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PASSIVE GEODETIC SATELLITE INFLATION RATE STUDY

by Ernesto Saleme

Prepared by
IIT RESEARCH INSTITUTE
Chicago, Ill.
for Langley Research Center

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SUMMARY

This study comprises the mechanics of the inflation process of a spherical balloon orbiting at high altitude. Mathematical models suitable for the two basic stages of the process (deployment and inflation) are developed, and numerical solutions using high speed digital computers are presented.

A method is also established for determining the stresses developed in the skin during the inflation process. The elasticity of the transverse (accordion) folds is analyzed by means of an equivalent spring, and that of the meridian pleats by using the large deformation theory of beams.

The conditions (modeling laws) that a model test must satisfy in order to accurately describe the behavior of the prototype are established. The effects of lack of compliance with these modeling laws are discussed.

Computer programs for the solution of the two stages of the process and several numerical examples are included.

INTRODUCTION

As part of the National Geodetic Satellite Program, a Passive Geodetic Satellite has been launched into a near polar orbit. The aluminum coated spherical satellite can be observed from the ground as a point source of light while it reflects the incident light. Simultaneous photographs of this light source taken from different points on the earth surface will permit the determination of the spatial coordinates of these points and, hence, with an adequate network of ground stations, a purely geometric determination of the shape and size of the earth can be obtained.

The inflatable sphere is fabricated by joining a number of gores of thin plastic film coated with vapor-deposited aluminum. During fabrication the material is pleat folded along meridian lines and then the whole assembly is placed in a long narrow plastic sleeve and evacuated. After evacuation the pleat folded balloon assembly is placed into a spherical canister by folding it in a rotating accordion pattern. The canister is then evacuated and sealed.

When the canister opens in space, it has been observed that the balloon deploys by opening the accordion folds and inflates until it assumes the final spherical shape. The first phase, balloon erection, is dominated by the deployment of the accordion folds while the final phase is dominated by the unfolding of the pleat folded gores by inflation. Between these two relatively simple phases of deployment and inflation there is a transition phase where both accordion folds and pleat folds are being unfolded.

Previous theoretical analyses of the erection process were based on a single stage spherical mathematical model. The present study has evolved a two stage model; the first stage is a deployment model and accounts for the deployment of the accordion folds; the second stage is an inflation model which accounts for the unfolding of the pleat folded gores.

The present two stage model does not account for the transition phase between deployment and inflation but instead considers sequentially the deployment and inflation stages. Justification for this procedure actually rests on the lack of an adequate mathematical model representing the transition stage. The actual deployment will be slower than predicted by the present two stage model but, on the other hand, the actual inflation will start sooner than predicted. Due to the compensating nature of these effects, the total erection time should be close to the predicted value. Likewise, the stresses, velocities, accelerations, etc., should be close to the predicted values except near the end of deployment and near the beginning of inflation.

Finally, the scaling laws that must be satisfied by a model tested on the ground are established in general terms.

THE INFLATION PROCESS

During fabrication, the material is pleat folded along meridian lines and afterwards folded again accordion wise in a transverse direction and placed into the canister.

When the canister opens in space, the folded balloon first deploys by opening up the transverse folds and assuming an elongated, cigar-like shape (Deployment Stage) and then inflates by opening the meridian pleats, assuming an ellipsoidal form with a star shaped cross section, which finally becomes a sphere (Inflation Stage).

Deployment Stage

This stage is characterized by a large increase in the polar diameter and a small variation in the transverse dimensions of the balloon. In order to analyze the mechanics of this stage, we make use of a mathematical model. A continuous structure in the shape of a shell of revolution about the polar axis is substituted for the accordion folded (sectionally continuous) balloon. The substitute structure is of the same total length and equatorial cross section as the actual balloon.

Moreover, we assume that, during this stage, the cross sections (normal to the direction of deployment) remain invariant, i.e. all points move parallel to the direction of deployment (polar axis) (Figure 1).

The mass between two parallel circles, a meridian distance ds apart will be

$$2\pi \, m_1 \, x_1 \, ds_1 = 2\pi \, m \, x \, ds$$

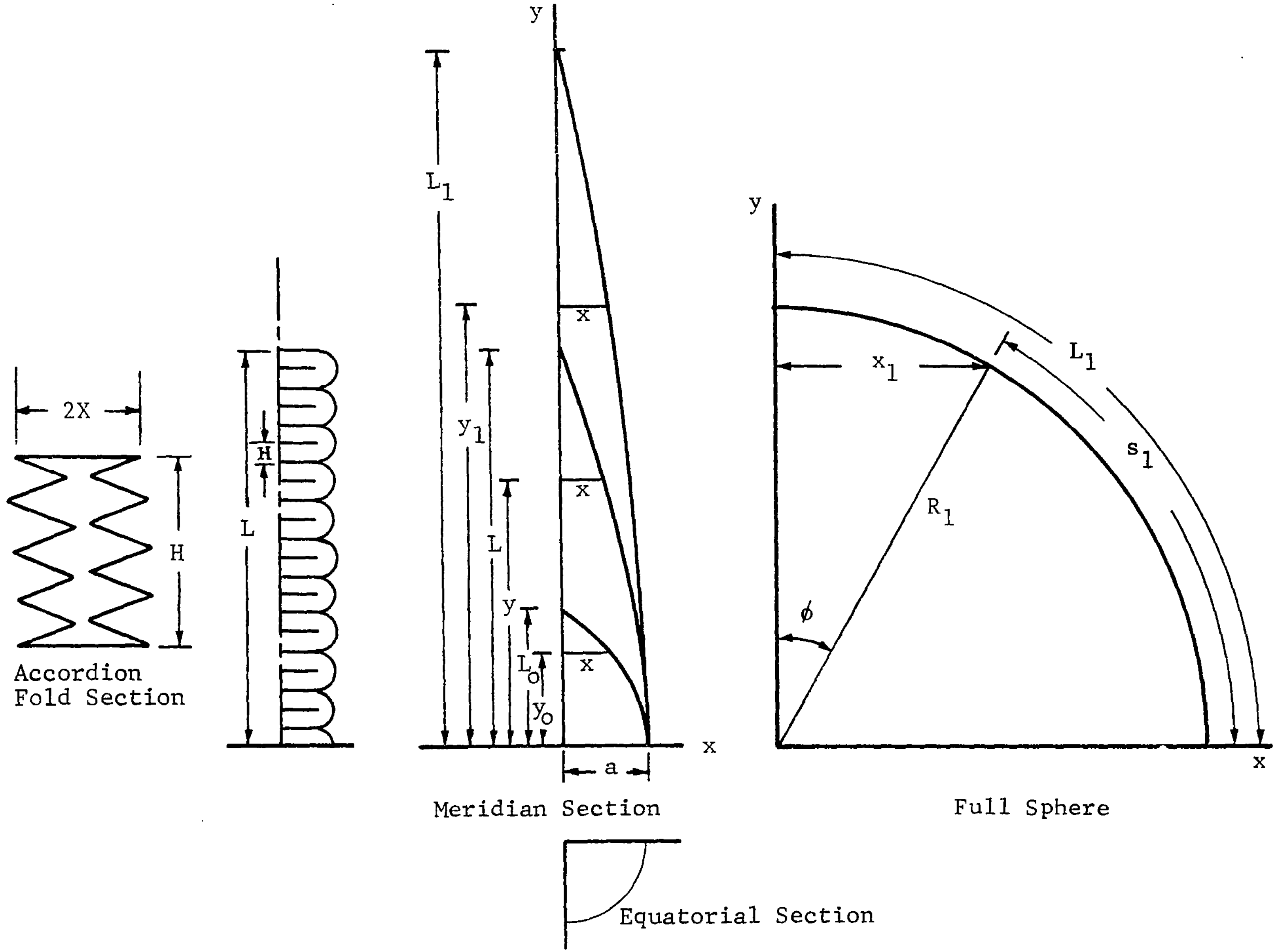


Figure 1 First Stage-Deployment

where

m_1, m are the masses per unit area of the full sphere and the deployment stage balloon respectively.

x_1, x are the radius of the parallel in the full sphere and the deployment stage balloon respectively.

$ds_1 = ds$ is the length of the element of meridian arc.

Hence

$$m = m_1 \frac{x_1}{x} \frac{ds_1}{ds} \tag{1}$$

Assuming that the meridian has a cosine shape

$$x = a \cos \frac{\pi}{2} \frac{y}{L} \tag{2}$$

where

a is the equatorial radius of the balloon during deployment.

y is the distance from the parallel circle to the equator.

L is the distance from the tip (pole) to the equator.

For a very elongated shape, as is the present case, the distance y will differ but little from the distance $\bar{s}$ measured along the meridian, i.e. we can set

$$\bar{s} = y \tag{3}$$

Hence, for the full sphere

$$x_1 = R_1 \cos \frac{\bar{s}_1}{R_1} = R_1 \cos \frac{\pi}{2} \frac{y_1}{L_1} \tag{4}$$

where $y_1 = \bar{s}_1$; $L_1 = \frac{\pi}{2} R_1$ are the distances to the equator, measured along the meridian, from the parallel circle and the pole in the fully deployed balloon. The assumption that the dimensions of the cross section do not change during deployment implies

$$x = a \cos \frac{\pi}{2} \frac{y}{L} = a \cos \frac{\pi}{2} \frac{y_1}{L_1} = a \cos \bar{\phi} \tag{5}$$

where

$$\bar{\phi} = \frac{\pi}{2}\frac{y}{L} = \frac{\pi}{2}\frac{y_1}{L_1} \tag{6}$$

is independent of time.

Equation (1) finally yields:

$$m = m_1 \frac{R_1 \cos\frac{\pi}{2}\frac{y_1}{L_1}}{a \cos\frac{\pi}{2}\frac{y}{L}} \frac{dy_1}{dy} = m_1 \frac{R_1}{a}\frac{L_1}{L} = \frac{\pi}{2} m_1 \frac{{R_1}^2}{aL} \tag{7}$$

The total mass of the balloon is given by

$$M = 4\pi m_1 {R_1}^2 \tag{8}$$

Hence

$$m = \frac{M}{8aL} \tag{9}$$

The differential equation of motion during deployment is obtained in the following way:

We consider an element of the balloon a distance y from the equator. The mass of the element will be

$$2\pi\ m\ x\ ds = \frac{\pi}{4}\frac{M}{L} \cos\frac{\pi}{2}\frac{y}{L}\, dy$$

and the inertia force will be

$$\bar{F}_1 = -\frac{\pi}{4}\frac{M}{L}\cos\frac{\pi}{2}\frac{y}{L}\, dy \frac{\partial^2 U(y,t)}{\partial t^2} \tag{10}$$

where $U(y,t)$ is the displacement and the minus sign indicates that the inertia force is opposed to the motion. We assume that the force due to the accordion folds is of the form

$$-F(y)\frac{\partial U(y,t)}{\partial y}$$

i.e. is proportional to the deformation $\frac{\partial U}{\partial y}$ (in the case of an elastic bar of constant cross section and small deformations $F(y)$

will be constant). The net force exerted by the accordion folds on the element will be then

$$\bar{F}_2 = -\frac{\partial}{\partial y} F(y) \frac{\partial U(y,t)}{\partial y} dy \tag{11}$$

The internal pressure, -p(V,T) is assumed to be uniform inside the deploying balloon and depending only on the volume and absolute temperature and the net force exerted by it on the element will be

$$\bar{F}_3 = - \frac{d}{dy} \pi x^2 p(V,T) dy = - 2\pi p(V,T) x \frac{dx}{dy} dy \tag{12}$$

The differential equation of motion is obtained by setting

$$\bar{F}_1 + \bar{F}_2 + \bar{F}_3 = 0$$

or

$$- \frac{\pi}{4} \frac{M}{L} \cos \frac{\pi}{2} \frac{y}{L} dy \frac{\partial^2 U(y,t)}{\partial t^2} - \frac{\partial}{\partial y} F(y) \frac{\partial U(y,t)}{\partial y} dy$$

$$- 2 \pi p(V,T) x \frac{dx}{dy} dy = 0$$

Using Equation (5) we obtain after simplification

$$\frac{\pi}{4} \frac{M}{L} \cos \bar{\phi} \frac{\partial^2 U(y,t)}{\partial t^2} + \frac{\partial}{\partial y} F(y) \frac{\partial U(y,t)}{\partial y}$$

$$- \frac{\pi^2 a^2}{L} p(V,T) \sin \bar{\phi} \cos \bar{\phi} = 0 \tag{13}$$

Comparing the first and last terms of Equation (13) we see that they will have the same form if we take U(y,t) to be:

$$U(y,t) = (L-L_o) \sin \frac{\pi}{2} \frac{y}{L} = (L-L_o) \sin \bar{\phi} \tag{14}$$

where $L=L(t)$ is half the length of the deployed balloon at time t and

$$L\ (t=0) = L_o$$

is the initial half length of the folded balloon (at the instant the canister opens). The displacement $U(y,t)$ is seen to satisfy the conditions

$$U(o,t) \equiv 0$$

$$U(y,o) = U(\bar{\phi},o) \equiv 0$$

Substituting Equation (14) into (13) we obtain:

$$\frac{M}{2} \sin \bar{\phi} \cos \bar{\phi} \frac{d^2L}{dt^2} + (L-L_o) \frac{d}{dy} F(y) \cos \bar{\phi}$$

$$- 2\pi\ a^2\ p(V,T) \sin \bar{\phi} \cos \bar{\phi} = 0 \qquad (15)$$

In Section IV it is shown that the force developed by the elasticity of the accordion folds can be expressed as:

$$F(y) = F(L) \cos \frac{\pi}{2} \frac{y}{L} = F(L) \cos \bar{\phi} \qquad (16)$$

Substituting Equation (16) into (15) we obtain, after simplification:

$$\ddot{L} = \frac{2\pi}{M} \left[(1 - \frac{L_o}{L}) F(L) + 2\ a^2\ p(V,T) \right] \qquad (17)$$

where the dots indicate differentiation with respect to time. Multiplying both sides of Equation (17) by

$$\dot{L}dt = dL$$

and integrating, we obtain:

$$\int_o^t \dot{L}\ \ddot{L}\ dt = \int_o^t \dot{L} \frac{d\dot{L}}{dt}\ dt = \frac{\dot{L}^2}{2} = \frac{2\pi}{M} \int_{L_o}^{L} \left[(1-\frac{L_o}{L}) F(L) + 2\ a^2 p(V,T) \right] dL \qquad (18)$$

or

$$\dot{L} = \frac{dL}{dt} = \sqrt{\frac{4\pi}{M} \int_{L_o}^{L} \left[(1-\frac{L_o}{L})\ F(L) + 2\ a^2\ p(V,T)\right]\ dL} \qquad (19)$$

From which we finally obtain

$$t = \int_{L_o}^{L} \frac{dL}{\sqrt{\frac{4\pi}{M} \int_{L_o}^{L} \left[(1-\frac{L_o}{L})\ F(L) + 2\ a^2\ p(V,T)\right]\ dL}} \qquad (20)$$

The internal pressure $p(V,T)$ is made up of two parts:

a) The pressure due to the residual gases left inside the balloon during fabrication is the first part. Assuming that they behave like perfect gases, their partial pressure will follow the law

$$\frac{P_r V}{T} = \frac{P_o V_o}{T_o} = \text{constant}$$

hence

$$p_r = p_o \frac{V_o}{V} \frac{T}{T_o} \qquad (21)$$

where

V_o, V are the initial and present volume enclosed by the balloon.

T_o, T are the initial and present values of the absolute temperature of the balloon.

p_o, p_r are the initial and present values of the pressure due to the residual gases.

b) The pressure due to sublimation of the chemical powders, if any, that are placed inside the balloon for the purpose of completing inflation and sustaining the spherical shape by providing the required internal pressure once the full inflation is attained, is the second part.

The corresponding partial pressure follows the Classius-Clapeyron equation

$$\log p_c = C_1 - \frac{C_2}{T} \tag{22}$$

where

p_c is the pressure generated by sublimation.

T is the absolute temperature.

C_1, C_2 are constants depending on the nature of the chemicals. The total pressure inside the balloon will be equal to the sum of the two partial pressures

$$p = p_r + p_c \tag{23}$$

In Appendix A it is shown that, for the chemicals and temperatures here considered, there will be only a small change in temperature during the whole process. Moreover, the rate at which heat is accumulated in the balloon during deployment is larger than the rate necessary to maintain the sublimation pressure. Hence, we can write

$$p_r = p_o \frac{V_o}{V}$$

$$p_c = \text{constant.}$$

Taking into account that during deployment the volume is proportional to the length we have finally

$$p = p_c + p_o \frac{L_o}{L} \tag{24}$$

In Section III, the following expression is derived for F(L)

$$F(L) = C_o \frac{f\left(\frac{L}{L_1}\right)}{L^2} \tag{25}$$

with

$$\begin{aligned} f\left(\frac{L}{L_1}\right) &= C_1 & 0 \le \frac{L}{L_1} \le \alpha \\ f\left(\frac{L}{L_1}\right) &= A - B\frac{L}{L_1} + C\left(\frac{L}{L_1}\right)^2 - D\left(\frac{L}{L_1}\right)^3 + E\left(\frac{L}{L_1}\right)^4 & \alpha \le \frac{L}{L_1} \le 1.0 \end{aligned} \tag{26}$$

where $C_o = \frac{\pi^3}{48} N^2 \pi ER_1 h_s^3$ $C = 7.05571611$

$C_1 = 0.29845520$ $D = 5.74280591$

$A = 0.49614662$ $E = 1.82244592$

$B = 2.63150274$ $\alpha = 0.45694658$

Substituting Equation (24) and (25) into Equation (17) we obtain, on account of Equation (26):

$$\ddot{L} = \frac{2\pi}{M}\left[C_o \frac{C_1}{L^2}\left(1 - \frac{L_o}{L}\right) + 2a^2\left(p_c + p_o \frac{L_o}{L}\right)\right] \quad 0 \le \frac{L}{L_1} \le \alpha$$

$$\ddot{L} = \frac{2\pi}{M}\left[C_o \frac{A - B\frac{L}{L_1} + C\left(\frac{L}{L_1}\right)^2 - D\left(\frac{L}{L_1}\right)^3 + E\left(\frac{L}{L_1}\right)^4}{L^2}\left(1 - \frac{L_o}{L_1}\right) + 2a^2\left(p_c + p_o\frac{L_o}{L}\right)\right] \quad \alpha \le \frac{L}{L_1} \le 1.0 \tag{27}$$

Multiplying both sides of Equation (17) by $2\dot{L}dt = 2dL$ and integrating we obtain finally:

$$\dot{L} = \sqrt{\frac{4\pi L_o}{M}}\left[\frac{C_o}{L_o^2}\frac{C_1}{2}\left(1 - \frac{L_o}{L}\right)^2 + 2a^2\left[p_c\left(\frac{L}{L_o} - 1\right) + p_o \log\frac{L}{L_o}\right]\right]^{1/2} \quad 0 \le \frac{L}{L_1} \le \alpha$$

$$\dot{L} = \sqrt{\frac{4\pi L_o}{M}}\left[\frac{C_o}{L_o^2}\left[\frac{C_1}{2}\left(1 - \frac{L_o}{\alpha L_1}\right)^2 - \frac{A}{2}\left(\frac{L_o}{L}\right)^2\left(\frac{L}{\alpha L_1}\right)^2 - 1\right] + \left(A + B\frac{L_o}{L_1}\right)\frac{L_o}{L}\left(\frac{L}{\alpha L_1} - 1\right) - \left(B + C\frac{L_o}{L_1}\right)\frac{L_o}{L_1}\log\frac{L}{\alpha L_1} + \left(C + D\frac{L_o}{L_1}\right)\frac{L_o}{L_1}\left(\frac{L}{L_1} - \alpha\right) - \frac{1}{2}\left(D + E\frac{L_o}{L_1}\right)\frac{L_o}{L_1}\left[\left(\frac{L}{L_1}\right)^2 - \alpha^2\right] + \frac{E}{3}\frac{L_o}{L_1}\left[\left(\frac{L}{L_1}\right)^3 - \alpha^3\right]\right]^{1/2} \quad \alpha \le \frac{L}{L_1} \le 1 \tag{28}$$

In Appendix B a computer program for the integration of Equation (20) using Equation (28) with provisions for the case when Equation (18) cannot be readily integrated is presented and numerical examples are worked out.

Inflation Stage

During the inflation stage the balloon increases its equatorial dimension and at the same time decreases its polar diameter. In this way the balloon passes from the elongated shape at the end of the deployment stage to the final spherical shape.

Referring to Figure 2, consider the dynamic equilibrium of an element of the shell, of area $x\, d\theta\, ds$, between two adjacent meridians and two parallel circles. The mass of the element will then be

$$m\, x\, d\theta\, ds$$

where

- m is the mass per unit area of the shell,
- x is the radius of the parallel circle,
- $d\theta$ is the angular distance between the meridians, and
- ds is the meridian distance between the parallel circles.

Let $\ddot{x}$, $\ddot{y}$ be the accelerations in the horizontal and vertical directions. Then the inertia forces will be

$$- m\, x\, d\theta\, ds\, \ddot{x} \tag{29}$$

$$- m\, y\, d\theta\, ds\, \ddot{y} \tag{30}$$

where the dots indicate differentiation with respect to time. Let F_θ be the membrane force per unit length of meridian. Then, forces of magnitude $F_\theta\, ds$ and direction tangent to the parallel circle will act on the lateral sides of the element. The resultant of these forces will be

$$F_\theta\, ds\, d\theta \tag{31}$$

directed towards the center of the parallel circle.

Let F_ϕ be the membrane force per unit length of parallel circle in a direction tangent to the meridian. A force will act on the upper side of the element

$$- F_\phi\, x\, d\theta \cos \phi$$

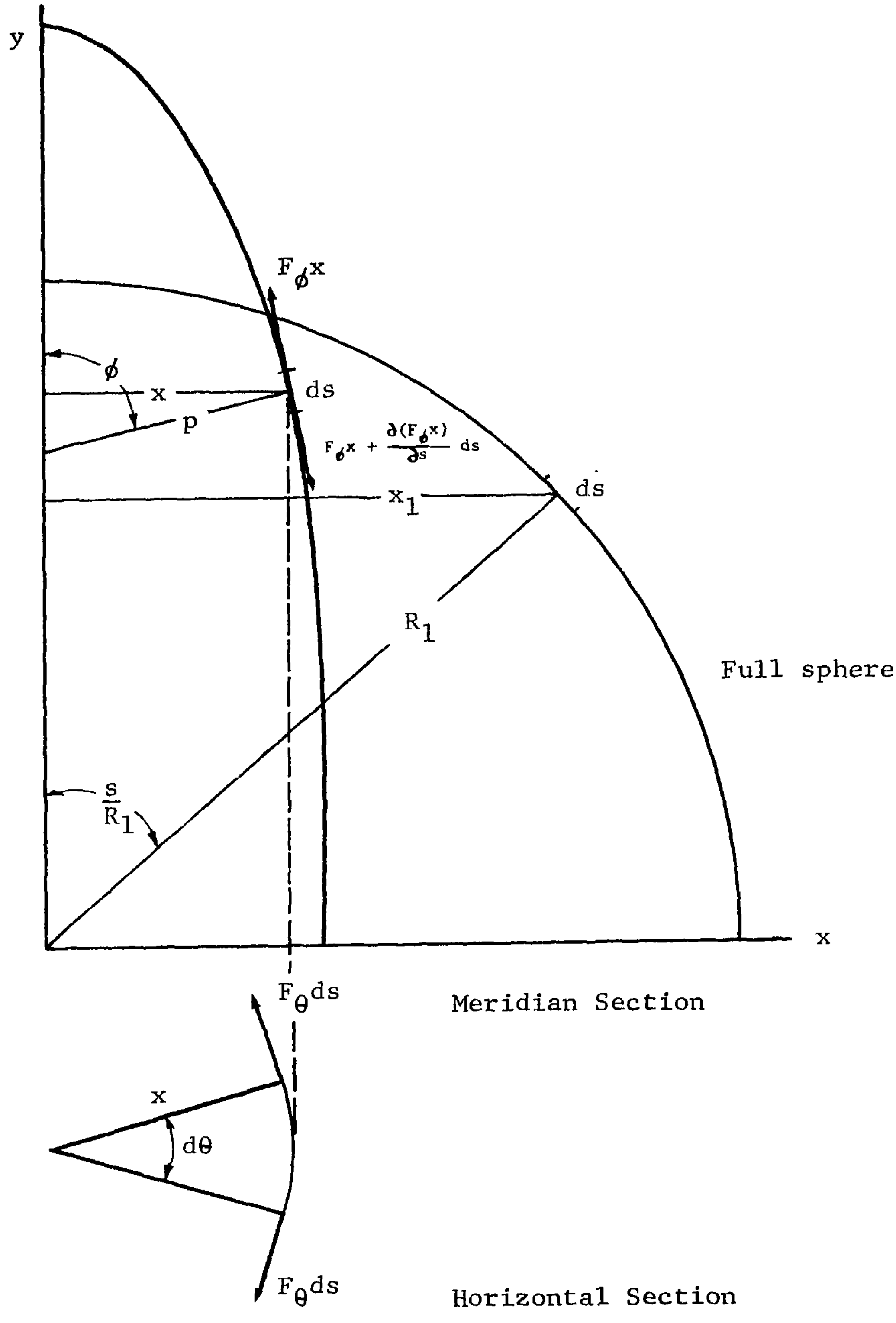


Figure 2 Second Stage-Inflation

in the horizontal direction and a force

$$F_\phi \, x \, d\theta \sin \phi$$

in the vertical direction.

On the lower side of the element we have

$$F_\phi \, x \, d\theta \cos \phi + \frac{\partial}{\partial s} (F_\phi \, x \, d\theta \cos \phi) \, ds$$

in the horizontal direction and

$$- \left[F_\phi \, x \, d\theta \sin \phi + \frac{\partial}{\partial s} (F_\phi \, x \, d\theta \sin \phi) \, ds \right]$$

in the vertical direction. The resultant of these forces will have components

$$\frac{\partial}{\partial s} (F_\phi \, x \cos \phi) \, d\theta \, ds \tag{32}$$

in the horizontal direction and

$$- \frac{\partial}{\partial s} (F_\phi \, x \sin \phi) \, d\theta \, ds \tag{33}$$

in the vertical direction.

The internal pressure p acting on the element gives components

$$p \, x \, d\theta \, ds \sin \phi \tag{34}$$

in the horizontal direction and

$$p \, x \, d\theta \, ds \cos \phi \tag{35}$$

in the vertical direction.

Combining Equations (29), (31), (32) and (34) and Equations (30), (33) and (35) we obtain after simplification

$$m x \ddot{x} = p \, x \sin \phi - F_\theta + \frac{\partial}{\partial s} (F_\phi \, x \cos \phi) \tag{36}$$

$$m x \ddot{y} = p \, x \cos \phi - \frac{\partial}{\partial s} (F_\phi \, x \sin \phi) \tag{37}$$

the differential equations of motion.

The total mass between two parallel circles remains constant during the process, hence:

$$m \cdot 2\pi \; x \; ds = m_1 \cdot 2\pi \; x_1 \; ds_1 = m_1 \cdot 2\pi \; R_1 \; \sin \frac{s_1}{R_1} \; ds_1 \qquad (38)$$

where

- m, m_1 are the mass per unit area in the balloon and the full sphere respectively,
- ds, ds_1 are the element of the meridian arc in the balloon and the full sphere,
- x, x_1 are the radius of parallel circle in the balloon and the full sphere, and
- R_1 is the radius of the full sphere.

Equations (36) and (37) can be written

$$\bar{m} \; \ddot{x} = p \; x \; \sin \phi - F_\theta + \frac{\partial}{\partial s} (F_\phi \; x \; \cos \phi) \qquad (39)$$

$$\bar{m} \; \ddot{y} = p \; x \; \cos \phi - \frac{\partial}{\partial s} (F_\phi \; x \; \sin \phi) \qquad (40)$$

where

$$\bar{m} = mx = m_1 \; R_1 \; \sin \frac{s}{R_1} \; \frac{ds_1}{ds} \qquad (41)$$

During this stage, the balloon offers little resistance to change in the circumferential direction (opening of the meridian pleats) while its deformation in the meridian direction requires stretching of the skin. We then may assume that there is no deformation in the meridian direction, that is the distance between two points along the meridian is invariant with respect to time, or

$$ds = \sqrt{dx^2 + dy^2} = ds_1 \qquad (42)$$

independent of time.

The assumption that the elastic strains can be neglected (as compared with the deformation due to inflation), implies that the length of a parallel circle cannot be larger than the length of the corresponding parallel in the full sphere, i.e.

$$x \leq x_1 \qquad (42')$$

Multiplying Equation (40) by ds and integrating, taking into account that

$$\cos \phi = \frac{dx}{ds}$$

we obtain

$$F_\phi \, x \sin \phi = p \frac{x^2}{2} - \int_o^s \bar{m} \, \ddot{y} \, ds \qquad (43)$$

Equation (43) is equivalent to the differential Equation (40) hence, the system of the differential Equations (39) and (40) can be substituted by the single differential equation

$$\bar{m} \, \ddot{x} = p \, x \sin \phi - F_\theta + \frac{d}{ds} (F_\phi \, x \cos \phi) \qquad (39')$$

where, by Equation (43)

$$F_\phi = \frac{1}{x \sin \phi} \left[p \frac{x^2}{2} - \int_o^s \bar{m} \, \ddot{y} \, ds \right] \qquad (44)$$

The hoop force F_θ due to the opening of the meridian pleats is a function of the radius x of the parallel circle

$$F_\theta = F(x) \qquad (45)$$

In Section III, an expression for F_θ as a function of the parallel radius is derived and is found that in general, it can be neglected.

The differential Equation (39) together with the definition Equations (44) and (45) and the constraint conditions Equations (42) and (42'), can be solved numerically by a step by step procedure in the following way.

Let us assume the problem solved up to a time $t=t_n$. In order to determine the position at $t=t_{n+1}=t_n+\Delta t_{n+1}$, we assume that the gas pressure p and the accelerations $\ddot{x}$ and $\ddot{y}$ remain constant during the time interval $t_n \leq t \leq t_{n+1}$

$$p = p_n = p\ (V_n, T_n) \quad \text{(e.g. } p_n = p_o \frac{V_o}{V_n}\text{ , Boyle's Law)} \tag{46}$$

$$\ddot{x} = \ddot{x}_{n+1} \tag{47}$$

$$\ddot{y} = \ddot{y}_{n+1} \tag{48}$$

where V_n, T_n are the volume and temperature at time t_n. By integrating Equation (47) and (48) with respect to time from t_n to t_{n+1} we obtain the values of the velocities and displacements at $t=t_{n+1}$

$$\dot{x}_{n+1} = \dot{x}_n + \ddot{x}_{n+1}\ \Delta t_{n+1} \tag{49}$$

$$\dot{y}_{n+1} = \dot{y}_n + \ddot{y}_{n+1}\ \Delta t_{n+1} \tag{50}$$

$$x_{n+1} = x_n + \dot{x}_n \Delta t_{n+1} + \ddot{x}_{n+1} \frac{\Delta t^2_{n+1}}{2} \tag{51}$$

$$y_{n+1} = y_n + \dot{y}_n \Delta t_{n+1} + \ddot{y}_{n+1} \frac{\Delta t^2_{n+1}}{2} \tag{52}$$

The values of $\ddot{x}_{n+1}$ and $\ddot{y}_{n+1}$ are obtained by the following iteration process:

1. Assuming a starting value $\ddot{x}^o_{n+1}$ for $\ddot{x}_{n+1}$, determine x^o_{n+1} from Equation (51)

$$x^o_{n+1} = x_n + \dot{x}_n \Delta t_{n+1} + \ddot{x}^o_{n+1} \frac{\Delta t^2_{n+1}}{2} \tag{a}$$

2. Substitute Equation (a) into Equation (45) to obtain:

$$F^o_{\theta_{n+1}} = F_\theta(x^o_{n+1}) \tag{b}$$

3. Determine $\cos \phi^o_{n+1}$, $\sin \phi^o_{n+1}$ by

$$\cos \phi^o_{n+1} = \frac{\partial x^o_{n+1}}{\partial s}$$

$$\sin \phi^o_{n+1} = \sqrt{1 - \left(\frac{\partial x^o_{n+1}}{\partial s}\right)^2} \tag{c}$$

4. Determine y^o_{n+1} from Equation (42)

$$y^o_{n+1} = \int_{\frac{\pi}{2}R_1}^{s} \sqrt{ds^2 - dx^{o\,2}_{n+1}} \tag{d}$$

5. Determine $\ddot{y}^o_{n+1}$ from Equation (52)

$$\ddot{y}^o_{n+1} = \frac{2}{\Delta t^2_{n+1}} \left[y^o_{n+1} - y_n - \dot{y}_n \Delta t_{n+1} \right] \tag{e}$$

6. Determine $F^o_{\phi_{n+1}}$ from Equation (44)

$$F^o_{\phi_{n+1}} = \frac{1}{x^o_{n+1} \sin \phi^o_{n+1}} \left[p_n \frac{x^{o\,2}_{n+1}}{2} - \int_o^s \bar{m}\, \ddot{y}^o_{n+1}\, ds \right] \tag{f}$$

7. Determine $\ddot{x}^1_{n+1}$ from Equation (39)

$$\ddot{x}^1_{n+1} = \frac{1}{\bar{m}} \left[p_n\, x^o_{n+1} \sin \phi^o_{n+1} - F^o_{\theta_{n+1}} + \frac{\partial}{\partial s} F^o_{\phi_{n+1}}\, x^o_{n+1} \sin \phi^o_{n+1} \right] \tag{g}$$

8. Repeat steps (1) through (7) until there is no change in the values of the parameters. In practice, the process is stopped when the difference between two successive values of a given parameter, say x, is less in absolute value than a prescribed amount.

ELASTICITY OF THE FOLDS

Accordion Folds

In order to evaluate the force F(y) due to the elasticity of the accordion folds, we assimilate the folded balloon to a spring so that F(y) is the spring force at a distance y from the equator when the total distance between the ends is 2L. Figure 3 shows the centerline of a general fold between two consecutive bends. Assuming that the cross section at the bends remains horizontal, the differential equation of the elastica can be written in the notation of Figure 4.

$$EJ_k \frac{d\phi}{ds} = P_k(\ell_k - x) \tag{53}$$

Assuming that the moment of inertia J remains constant along the fold, we obtain by differentiation:

$$EJ \frac{d^2\phi}{ds^2} = - P \frac{dx}{ds} = - P\sin\phi \tag{54}$$

where we dropped the index k. Multiplying both sides of Equation (54) by $d\phi = \frac{d\phi}{ds} ds$ and integrating we get:

$$\left(\frac{d\phi}{ds}\right)^2 = \frac{2P}{EJ}\left[\cos\phi - \cos\phi_\ell\right]$$

From which

$$ds = \sqrt{\frac{EJ}{2P}} \frac{d\phi}{\sqrt{\cos\phi - \cos\phi_\ell}} \tag{55}$$

Integration of Equation (55) yields

$$\ell = \int_o^\ell ds = \sqrt{\frac{EJ}{2P}} \int_o^{\phi_\ell} \frac{d\phi}{\sqrt{\cos\phi - \cos\phi_\ell}} \tag{56}$$

Let

$$\sin\frac{\phi}{2} = k \sin\theta$$
$$k = \sin\frac{\phi_\ell}{2} \tag{57}$$

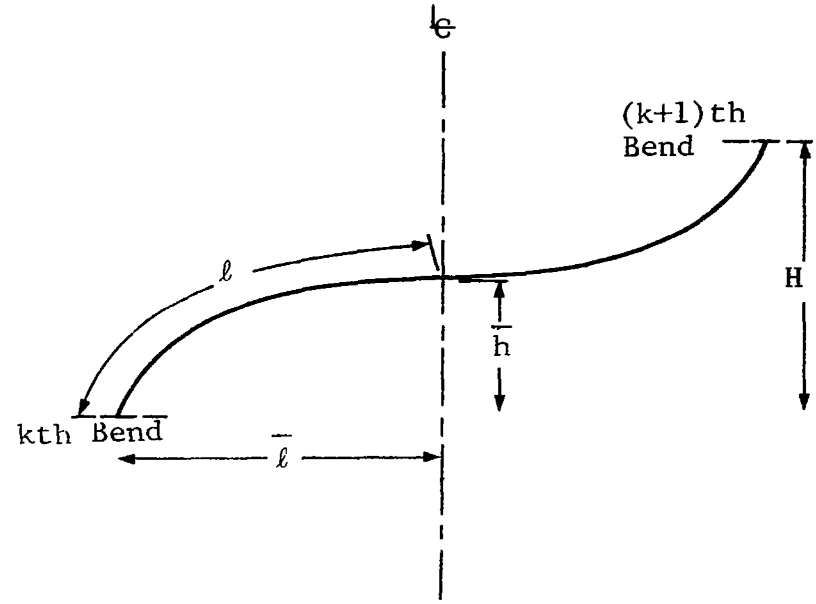


Figure 3 Centerline of an Accordion Fold

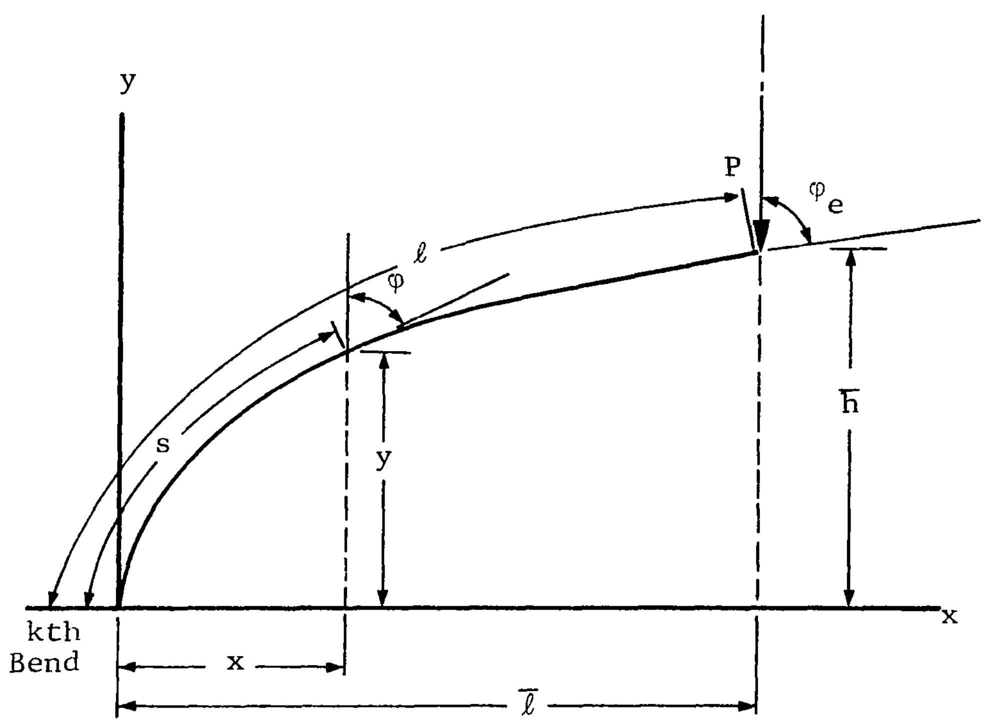


Figure 4 Equivalent Cantilever Beam

Substituting Equation (57) into (56) we obtain:

$$\ell = \sqrt{\frac{EJ}{P}} \int_0^{\frac{\pi}{2}} \frac{d\theta}{\sqrt{1-k^2 \sin^2\theta}} = \sqrt{\frac{EJ}{P}}\, K(k) \tag{58}$$

where K(k) is the complete elliptic integral of the first kind. Multiplying both sides of Equation (55) by $\cos\phi$ we obtain:

$$dy = ds \cos\phi = \sqrt{\frac{EJ}{2P}} \frac{\cos\phi \; d\phi}{\sqrt{\cos\phi - \cos\phi_\ell}}$$

Integrating we get

$$\bar{h} = \int_0^{\bar{h}} dy = \int_0^{\ell} ds \cos\phi = \sqrt{\frac{EJ}{2P}} \int_0^{\phi_\ell} \frac{\cos\phi \; d\phi}{\sqrt{\cos\phi - \cos\phi_\ell}} \; . \tag{59}$$

Which, by Equation (57), becomes

$$\bar{h} = \sqrt{\frac{EJ}{P}} \int_0^{\frac{\pi}{2}} \frac{(1-2k^2 \sin^2\theta)\; d\theta}{\sqrt{1-k^2 \sin^2\theta}} = \sqrt{\frac{EJ}{P}} \left[2E(k) - K(k)\right] \tag{60}$$

Where E(k) is the complete elliptic integral of the second kind. From Equation (58) and (60) we obtain:

$$\frac{\bar{h}}{\ell} = 2 \frac{E(k)}{K(k)} - 1 \tag{61}$$

Assuming all the folds to have the same height and the same length, we have

$$\begin{aligned} \bar{h} &= \frac{H}{2} = \frac{1}{2} \; \frac{2L}{N} \\ \ell &= \frac{1}{2} \; \frac{2L_1}{N} \\ \frac{\bar{h}}{\ell} &= \frac{L}{L_1} \end{aligned} \tag{62}$$

By plotting $\frac{P\bar{h}^2}{EJ}$ against $\frac{\bar{h}}{\ell}$ for various values of k, it was found that P could be expressed as

$$P = \frac{\pi^2 N^2 EJ}{4\ L^2}\ f\left(\frac{L}{L_1}\right) \tag{63}$$

Where $f(\frac{L}{L_1})$ is given approximately by:

$$f\left(\frac{L}{L_1}\right) = 0.29845520 \qquad \left(0 \le \frac{L}{L_1} \le 0.45694658\right)$$

$$f\left(\frac{L}{L_1}\right) = 0.49614662 - 2.63150274\left(\frac{L}{L_1}\right) + 7.05571611\left(\frac{L}{L_1}\right)^2$$
$$-5.74280591\left(\frac{L}{L_1}\right)^3 + 1.82244592\left(\frac{L}{L_1}\right)^4$$
$$\left(0.45694658 \le \frac{L}{L_1} \le 1.0\right) \tag{64}$$

The moment of inertia J_k of the kth fold can be taken as:

$$J_k = J_o \cos \frac{s_k}{R_1}$$

Where

J_o is the moment of inertia of the equatorial cross section.

S_k is the distance, measured along the folded baloon, from the center of the kth fold to the equator.

In view of the large number of folds, we may substitute the discrete distribution of the moments of inertia by a continuous one and write

$$J(s) = J_o \cos \frac{s}{R_1} = J_o \cos \frac{\pi}{2} \frac{y}{L} \tag{65}$$

The moment of inertia J_o at the equatorial cross section is given by:

$$J_o = \left(\frac{n}{2}\right)^2 \frac{2a_1}{12} h_s^3 = \frac{n^2}{24} a_1 h_s^3 = \frac{\pi}{12} nR_1 h_s^3 \tag{66}$$

where

n = number of meridian pleat folds.

$a_1 = \frac{2\pi R_1}{n}$ = equatorial width of the meridian pleat folds

h_s = skin thickness.

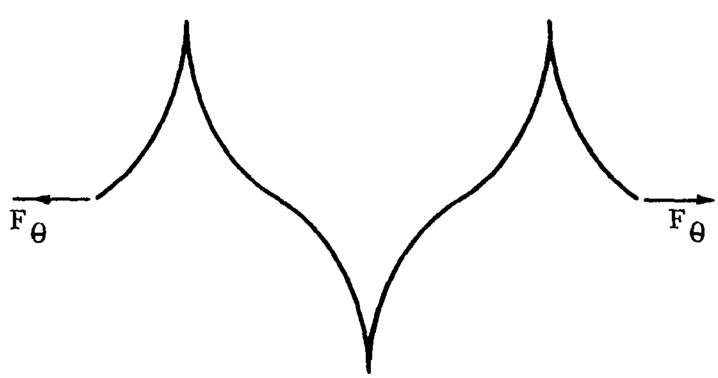
Making $F(y) = P$ of Equation (63) and using Equation (64) and (65) we obtain finally

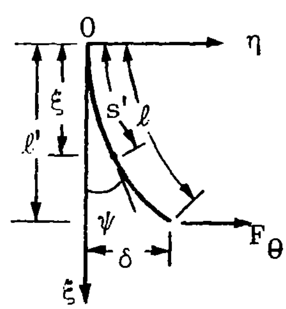
$$F(y) = \frac{\pi^3}{48} N^2 n \; E R_1 h_s^3 \; \frac{f\left(\frac{L}{L_1}\right)}{L^2} \cos \frac{\pi}{2} \frac{y}{L} = F(L) \cos \frac{\pi}{2} \frac{y}{L} \qquad (67)$$

as the force due to the elasticity of the accordion folds.

Meridian Pleats

During fabrication the balloon is folded along meridian lines and placed inside a plastic sleeve which is then evacuated. As a consequence the meridian pleats offer a relatively high resistance to opening, while on the other hand, the thin plastic skin offers very little resistance to bending. Hence we may represent the skin between pleat folds by a very flexible beam whose ends are subjected to parallel displacement with respect to each other.





On account of symmetry, it is sufficient to consider one-half of the skin between two pleats.

The differential equation of the elastica for the resulting cantilever beam can be written

$$\frac{EI}{\rho} = EI \frac{d\psi}{ds'} = F_\theta \cdot (\ell' - \xi) \tag{68}$$

where E is the modulus of elasticity of the material

$I = \frac{h_s^3}{12}$ is the moment of inertia of the skin per unit length of meridian

F_θ is the hoop force per unit length of meridian

ρ is the radius of curvature of the elastica at the point

ψ is the angle that the tangent to the elastica makes with the ξ-axis

s' is the distance from the origin to the point, measured along the elastica

ξ is the abscissa of the point in the deformed state

ℓ' is the abscissa of the mid-point in the deformed state

Differentiating Equation (68) with respect to s' we obtain

$$\frac{d^2\psi}{ds'^2} = -\beta^2 \frac{d\xi}{ds'} = -\beta^2 \cos\psi \tag{69}$$

where $\beta^2 = \frac{F_\theta}{EI}$

Multiplying both sides of Equation (69) by $\frac{d\psi}{ds'}\, ds'$ and integrating we obtain

$$\frac{1}{2}\left(\frac{d\psi}{ds'}\right)^2 = \beta^2 (\sin\psi_\ell - \sin\psi)$$

or

$$ds' = \frac{1}{\beta\sqrt{2}} \frac{d\psi}{\sqrt{\sin\psi_\ell - \sin\psi}} \tag{70}$$

Where $\tan \psi_\ell$ is the slope at the end of the cantilever. Integration of Equation (70) yields

$$\ell = \frac{1}{\beta\sqrt{2}} \int_0^{\psi_\ell} \frac{d\psi}{\sqrt{\sin\psi_\ell - \sin\psi}} \tag{71}$$

where 2ℓ is the distance between pleat folds.

Let

$$\begin{aligned} 1 + \sin\psi_\ell &= 2k^2 \\ 1 + \sin\psi &= 2k^2 \sin^2\theta \end{aligned} \tag{72}$$

By Equation (72) Equation (71) yields

$$\beta\ell = \int_{\theta_o}^{\frac{\pi}{2}} \frac{d\theta}{\sqrt{1-k^2\sin^2\theta}} = F(k, \tfrac{\pi}{2}) - F(k,\theta_o) \tag{73}$$

where $F(\ )$ is the elliptic integral of the first kind and

$$\theta_o = \sin^{-1} \frac{1}{k\sqrt{2}}$$

Multiplying both sides of equation (70) by $\sin\psi$ we obtain

$$ds' \sin\psi = d\eta = \frac{1}{\beta\sqrt{2}} \frac{\sin\psi \ d\psi}{\sqrt{\sin\psi_\ell - \sin\psi}} \tag{74}$$

Integration of Equation (74) yields for the end deflection:

$$\delta = \frac{1}{\beta\sqrt{2}} \int_o^{\psi_\ell} \frac{\sin\psi \ d\psi}{\sqrt{\sin\psi_\ell - \sin\psi}} = \frac{1}{\beta} \int_{\theta_o}^{\frac{\pi}{2}} \frac{2k^2 \sin\psi - 1}{\sqrt{1-k^2\sin^2\psi}} d\psi$$

Finally

$$\delta = \frac{1}{\beta} \left[F(k, \tfrac{\pi}{2}) - F(k,\theta_o) - 2 \left[E(k, \tfrac{\pi}{2}) - E(k,\theta_o) \right] \right] \tag{75}$$

where $E(\ )$ is the elliptic integral of the second kind.

By Equation (73) Equation (75) can be written

$$\frac{\delta}{\ell} = 1 - 2 \frac{E(k, \frac{\pi}{2}) - E(k,\theta_o)}{F(k, \frac{\pi}{2}) - F(k,\theta_o)} \tag{76}$$

From Equations (68) and (70) we obtain, at $\xi=0$, $\psi=0$

$$\frac{\ell'}{\ell} = \frac{\sqrt{2 \sin\psi_\ell}}{\beta\ell} = \frac{\sqrt{2(2k^2 - 1)}}{F(k,\frac{\pi}{2}) - F(k,\theta_o)} \tag{77}$$

Equations (73), (76) and (77) give $\beta\ell = \sqrt{\frac{F_\theta \ell^2}{EI}}$, δ/ℓ and ℓ'/ℓ as functions of the parameter k. By plotting $F_\theta \ell^2/EI$ against δ/ℓ for various values of k it was found that F_θ can be expressed, approximately, as

$$F_\theta = 3\frac{EI}{\ell^2} \frac{\frac{\delta}{\ell}}{1 - 1.15\,(\frac{\delta}{\ell})^2} \qquad 0 \leq \frac{\delta}{\ell} < 0.9$$

$$F_\theta = \frac{EI}{0.12\ell^2} \quad \frac{\frac{\delta}{\ell}}{1 - (\frac{\delta}{\ell})^2} \qquad 0.9 < \frac{\delta}{\ell} < 1.0 \tag{78}$$

Taking into account that

$$\delta = \frac{\pi x}{n_1} \qquad \ell = \frac{\pi x_1}{n_1}$$

where

- x, x_1 are the radii of the parallel circle of the balloon during inflation and in the final sphere respectively
- n_1 is the number of meridian pleats

Equation (78) can be written

$$F_\theta = 3\left(\frac{n_1}{\pi x_1}\right)^2 EI \frac{\frac{x}{x_1}}{1 - 1.15\,(\frac{x}{x_1})^2} \qquad 0 \leq \frac{x}{x_1} < 0.9$$

$$F_\theta = \left(\frac{n_1}{\pi x_1}\right)^2 \frac{EI}{0.12} \frac{\frac{x}{x_1}}{1 - (\frac{x}{x_1})^2} \qquad 0.9 \leq \frac{x}{x_1} < 1.0 \tag{79}$$

A sample calculation is carried out below for PAGEOS with the following data.

$E = 6.6 \times 10^5$ lb in.$^{-2}$ $= 6.6 \times 10^5 \times 6.8947 \times 10^4 = 4.55 \times 10^{10}$ dy cm^{-2}

$h_s = 0.5$ mils $= 0.5 \times 10^{-3} \times 2.54 = 1.27 \times 10^{-3}$ cm

$$I = \frac{h_s^{\,3}}{12} = 1.7 \times 10^{-10} \text{ cm}^4$$

For the 80 degree parallel

$$x_1 = R_1 \cos 80 \text{ deg} = \frac{100}{2} \times 30.48 \times 1736 = 264.6 \text{ cm}$$

$$n_1 = 418$$

$$\ell = \pi \frac{x_1}{n_1} = 1.99 \text{ cm}$$

Assuming 95 percent inflation, i.e. $x/x_1 = 0.95$ we have, using the second of Equations (79)

$$F_\theta = \frac{4.55 \times 10^{10} \times 1.7 \times 10^{-10}}{0.12 \times 1.99^2} \frac{0.95}{1-0.95^2} = 155 \text{ dy cm}^{-1}$$

$$= 155 \times 5.71 \times 10^{-6} = 8.85 \times 10^{-4} \text{ lb in.}^{-1}$$

and the corresponding hoop stresses will be

$$\sigma_\theta = \frac{F_\theta}{h_s} = \frac{8.85 \times 10^{-4}}{0.5 \times 10^{-3}} = 1.77 \text{ lb in.}^{-2}$$

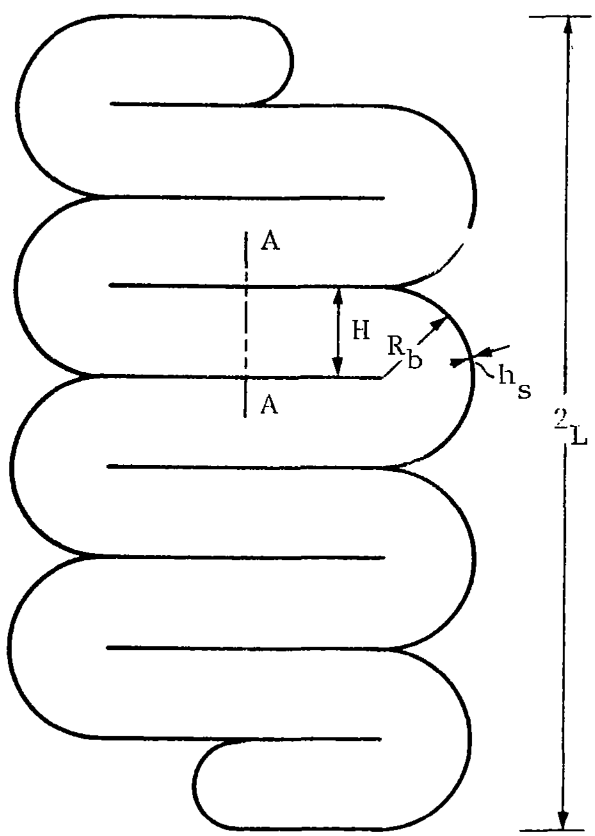
It can be seen that to assume $F_\theta \equiv 0$ will not affect appreciably the results.

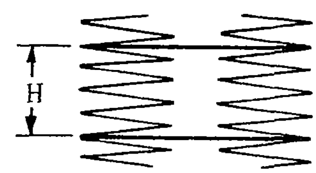
The above derivation has been based on the assumption that the axial deformations of the cantilever are identically zero. It is evident that this assumption cannot hold for very large deformations (δ/ℓ very close to unity) as F_θ becomes infinite for $\delta/\ell = 1$.

SKIN STRESSES

Deployment Stage

During the deployment stage, the height of the accordion folds increases by partial opening of the meridian pleats while the length of the parallel and meridian circles remain constant.





Section A-A

Considering a section A-A through an accordion fold we have for the membrane stresses the expression

$$\sigma_1 = \frac{pH}{2h_s} \tag{80}$$

where p is the internal pressure and h_s the skin thickness. The height H of an accordion fold can be taken approximately as

$$H = R_b = \frac{2L}{N} \tag{81}$$

where

R_b is the outside radius of the bend

$2L$ is the total length of the deployed balloon at time t

N is the number of accordion folds.

The foregoing derivation was based on the assumption that the accordion folds remained in contact, i.e.

$$\ell > \frac{\pi R_b}{2} = \frac{\pi L}{N} \tag{82}$$

where ℓ is the length of the fold.

Assuming all the folds to be of the same length

$$\ell = \frac{\pi}{2} \frac{D_1}{N} \tag{83}$$

where D_1 is the sphere diameter. Substituting Equation (83) into (82) we obtain

$$2L < D_1 \tag{84}$$

Hence, Equation (80) becomes

$$\sigma_1 = \frac{pL}{Nh_s} \qquad (2L < D_1) \tag{80'}$$

Consider now a section of the deploying balloon by a plane parallel to the direction of the accordion folds. The hoop stresses will be given by

$$\sigma_2 = \frac{p \cdot 2x\, dy}{2h_s\, d\bar{s}_1} \tag{85}$$

where

$2x$ is the diameter of the deploying balloon at the point

dy is the distance between two adjacent parallel circles

$d\bar{s}_1$ is the meridian distance between the parallel circles in the full sphere.

By Equations (2), (3) and (6), Equation (85) yields

$$\sigma_2 = \frac{pa}{h_s} \frac{L}{L_1} \cos \bar{\phi} \tag{86}$$

Finally, the membrane stresses developed in a section normal to the direction of deployment will be

$$\sigma_3 = \frac{p \cdot \pi x^2}{2\pi x_1 h_s} = \frac{px^2}{2x_1 h_s} \tag{87}$$

which, by Equations (2), (4) and (6) can be written

$$\sigma_3 = \frac{pa}{2h_s} \frac{L_o}{L_1} \cos \bar{\phi} \tag{88}$$

The computer program prints the values of the hoop stresses σ_2 given by Equation (86) at the Equator $(\bar{\phi} = 0)$.

Inflation Stage

The hoop stresses during the inflation stage are given by

$$\sigma_\theta = \frac{F_\theta}{h_s} \tag{89}$$

where F_θ is given by Equation (79). As shown in the sample calculation of Section III, the values of σ_θ will, during most of the inflation process, be very small.

The meridian stresses are given by:

$$\sigma_\phi = \frac{2\pi x F_\phi}{2\pi x_1 h_s} = \frac{xF_\phi}{x_1 h_s} \tag{90}$$

The computer program developed in Appendix B gives the values of the meridian stresses for a series of selected points along the meridian at various stages of inflation.

Stresses at the End of Deployment

At the end of the deployment stage, the coordinates and velocities of the points in the skin satisfy the relations

$$x = a \cos \frac{\pi}{2} \frac{y_1}{L_1} = a \cos \frac{y_1}{R_1}$$

$$\dot{y}_1 = \dot{L}_1 \sin \frac{\pi}{2} \frac{y_1}{L_1} = \dot{L}_1 \sin \frac{y_1}{R_1}$$

The elastic force acting on a cross section a distance y_1 from the equator will be

$$-F(y_1) \frac{\partial U}{\partial y_1} = - E\, 2\pi x_1 h_s \frac{\partial U_1}{\partial y_1} = - 2\pi\, Eh_s R_1 \cos \frac{y_1}{R_1} \frac{\partial U_1}{\partial y_1}$$

where E is the modulus of elasticity of the skin and $U_1 = U_1 (y_1, t)$ is the displacement from the equilibrium configuration $x = a\cos y_1/R_1$, y_1. The net force exacted on an element of length dy_1 will be:

$$\overline{F}_2 = - \frac{\partial}{\partial y} F(y) \frac{\partial U_1}{\partial y} = - 2\pi Eh_s R_1 \frac{\partial}{\partial y} \cos \frac{y_1}{R_1} \frac{\partial U_1}{\partial y} \qquad (11')$$

By Equation (10), (11') and (12), the differential equation of motion becomes:

$$- \frac{1}{2} \frac{M}{R_1} \cos \frac{y_1}{R_1} dy_1 \frac{\partial^2 U_1(y_1,t)}{dt^2} + 2\pi Eh_s R_1 \frac{\partial}{\partial y} \cos \frac{y_1}{R_1} \frac{\partial U_1(y_1,t)}{\partial y_1} dy_1 + 2\pi p_1 \frac{a^2}{R_1} \cos \frac{y_1}{R_1} \sin \frac{y_1}{R_1} dy_1 = 0 \qquad (13')$$

Where p_1 is the internal pressure at the end of deployment. Assuming $U_1(y_1,t)$ to be of the form:

$$U_1(y_1,t) = \overline{U}_1(t) \sin \frac{y_1}{R_1}$$

Equation (13') becomes after simplification

$$\ddot{\overline{U}}_1(t) + \frac{4\pi Eh_s}{M} \overline{U}_1(t) = \frac{4\pi a^2}{M} p_1$$

whose solution is

$$\overline{U}_1(t) = \frac{a^2}{Eh_s} p_1 + A \cos\sqrt{\frac{4\pi Eh_s}{M}}\, t + B \sin\sqrt{\frac{4\pi Eh_s}{M}}\, t$$

The constants A and B are determined from the initial conditions

$$U_1(y_1, t=0) = 0$$

$$\dot{U}_1(y_1, t=0) = \dot{y}_1 = \dot{L}_1 \sin\frac{y_1}{R_1}$$

yielding

$$A = -\frac{p_1 a^2}{Eh_s}$$

$$B = \sqrt{\frac{M}{4\pi Eh_s}}\, \dot{L}$$

Finally

$$U_1(y_1, t) = \left[\frac{p_1 a^2}{Eh_s}\left(1-\cos\sqrt{\frac{4\pi Eh_s}{M}}\, t\right) + \sqrt{\frac{M}{4\pi Eh_s}}\, \dot{L} \sin\sqrt{\frac{4\pi Eh_s}{M}}\, t\right]$$

$$\sin\frac{y_1}{R_1}$$

The longitudinal stresses are given by:

$$\sigma_\phi(y_1, t) = E\,\varepsilon\,(y_1, t) = E\,\frac{\partial U_1(y_1, t)}{\partial y} = \frac{E}{R_1}\left[\frac{p_1 a^2}{Eh_s}\left(1-\cos\sqrt{\frac{4\pi Eh_s}{M}}\, t\right) + \sqrt{\frac{M}{4\pi Eh_s}}\, \dot{L} \sin\sqrt{\frac{4\pi Eh_s}{M}}\, t\right] \cos\frac{y_1}{R_1}$$

The maximum value will occur at the equator ($y_1 = 0$):

$$\sigma_\phi(0, t) = \frac{E}{R_1}\left[\frac{p_1 a^2}{Eh_s}\left(1-\cos\sqrt{\frac{4\pi Eh_s}{M}}\, t\right) + \right.$$

$$\left. + \sqrt{\frac{M}{4\pi Eh_s}}\, \dot{L} \sin\sqrt{\frac{4\pi Eh_s}{M}}\, t\right]$$

Stresses at the end of the inflation. - At the end of the inflation stage, the points in the balloon surface have reached the spherical surface with a certain velocity. The stress analysis of the skin can now be carried on the assumption of small deflections and linear elasticity. The displacement components satisfy the differential equations:

$$\left[\frac{\partial^2}{\partial\phi^2} + \cos\phi\,\frac{\partial}{\partial\phi} - \left(\cot^2\phi + \nu + \frac{m_s R_1^2}{D}\,\frac{\partial^2}{\partial t^2}\right)\right]u + (1+\nu)\,\frac{\partial}{\partial\phi}\,w = 0$$

$$(1+\nu)\left[\frac{\partial}{\partial\phi} + \cot\phi\right]u + \left[2(1+\nu) + \frac{m_s R_1^2}{D}\,\frac{\partial^2}{\partial t^2}\right]w = 0 \qquad (91)$$

where

u, w	are the displacement components in the meridian and radial direction respectively.
$D = \frac{Eh_s}{1-\nu^2}$	is the membrane rigidity of the skin.
ν	is Poisson's ratio for the skin material.
R_1, ϕ	are the radius of the sphere and collatitude angle respectively.
p	is the pressure inside the balloon at the end of inflation.
$m_s = \frac{M}{4\pi R_1^2}$	is the mass per unit area of the skin.

The solution of Equation (91) is

$$u = \sum U_n P_n'(\phi)\,\sin\omega_n t$$

$$w = \frac{1-\nu}{2}\,\frac{PR_1^2}{Eh_s} + \sum W_n P_n(\phi)\,\sin\omega_n t \qquad (92)$$

where $P_n(\ )$ is the Legendre polynomial of the first kind of order n and the prima denotes differentiation with respect to ϕ. Substituting u and w into Equation (91) we obtain:

$$\left[\Omega_n^2 - n(n+1) + 1-\nu\right] U_n + (1+\nu) W_n = 0$$
$$n(n+1)(1+\nu) U_n + \left[\Omega_n^2 - 2(1+\nu)\right] W_n = 0 \tag{93}$$

where

$$\Omega_n^2 = \frac{m_s R_1^2}{D} \omega_n^2 \tag{94}$$

The requirement that Equation (93) have nontrivial solutions, leads to the frequency equation:

$$\Omega_n^4 - \left[n(n+1) + 1-3\nu\right]\Omega_n^2 + (1-\nu^2)(n-1)(n+2) = 0 \tag{95}$$

The solution of Equation (95) is

$$\Omega_n^2 = \frac{(n-1)(n+2)}{2}\left[1 + \frac{3(1+\nu)}{(n-1)(n+2)} \left[1 + 2\,\frac{(1+2\nu)(1+\nu)}{(n-1)(n+2)} + \left(3\,\frac{1+\nu}{(n-1)(n+2)}\right)^2\right]^{\frac{1}{2}}\right] \tag{96}$$

Differentiating Equation (92) with respect to time, we obtain the velocity components

$$\dot{u}(\phi,t) = \sum U_n \omega_n P'_n(\phi) \cos\omega t$$
$$\dot{w}(\phi,t) = \sum W_n \omega_n P_n(\phi) \cos\omega t \tag{97}$$

The symmetry of the problem with respect to the equatorial plane requires that n be even. Considering only the first two modes we have:

$$u(\phi,t) = -\frac{3}{2} U_2 \sin 2\phi \sin\omega_2 t$$
$$w(\phi,t) = \frac{1-\nu}{2}\,\frac{PR_1^2}{Eh_s} + w_o \sin\omega_o t + \frac{w_2}{4}(3\cos 2\phi + 1)\sin\omega_2 t \tag{98}$$

$$\dot{u}(\phi,t) = -\frac{3}{2} U_2\omega_2 \sin 2\phi \cos\omega_2 t$$

$$\dot{w}(\phi,t) = W_o\omega_o \cos\omega_o t + \frac{W_2\omega_2}{4} (3\cos 2\phi + 1) \cos\omega_2 t \qquad (99)$$

$$\omega_o^2 = \frac{2D}{m_s R_1^2}(1+\nu)$$

$$\omega_2^2 = \frac{2D}{m_s R_1^2}\left[\frac{7+3\nu}{4} \pm \sqrt{\left(\frac{7+3\nu}{4}\right)^2 - (1-\nu^2)}\right] \qquad (100)$$

The stresses are given by:

$$\sigma_\phi = \frac{pR_1^2}{2h_s} + \frac{E}{(1-\nu^2)R_1}\left[(1+\nu) W_o \sin\omega_o t + \left[\frac{1+\nu}{4} W_2 (3\cos 2\phi + 1) - 3U_2 (\cos 2\phi + \nu\cos^2\phi)\right] \sin\omega_2 t\right]$$

$$\sigma_\theta = \frac{pR_1^2}{2h_s} + \frac{E}{(1-\nu^2)R_1}\left[(1+\nu) W_o \sin\omega_o t + \left[\frac{1+\nu}{4} W_2 (3\cos 2\phi + 1) - 3U_2 (\cos 2\phi + \nu\cos^2\phi)\right] \sin\omega_2 t\right] \qquad (101)$$

$$\sigma_\theta = \frac{pR_1^2}{2h_s} + \frac{E}{(1-\nu^2)R_1}\left[(1+\nu) W_o \sin\omega_o t + \left[\frac{1+\nu}{4} W_2 (3\cos 2\phi + 1) - 3U_2 (\cos^2\phi + \nu\cos 2\phi)\right] \sin\omega_2 t\right]$$

By equating to zero the derivative of the stress with respect to ϕ we find that their maximum (or minimum) will occur at the pole ($\phi = 0$) or the equator ($\phi = \pi/2$). At the pole we have

$$\sigma_\phi = \sigma_\theta = \frac{pR_1}{2h_s} + \frac{E}{(1-\nu)R_1}\left[W_o \sin\omega_o t + (W_2 - 3U_2)\sin\omega_2 t\right] \qquad (102)$$

while at the equator

$$\sigma_\phi = \frac{pR_1}{2h_s} + \frac{E}{(1-\nu^2)\ R_1}\left[(1+\nu)\ w_o \sin\omega_o t - \left(\frac{1+\nu}{2} w_2 - 3U_2\right) \sin\omega_2 t\right]$$

$$\sigma_\theta = \frac{pR_1}{2h_s} + \frac{E}{(1-\nu^2)\ R_1}\left[(1+\nu)\ w_o \sin\omega_o t - \left(\frac{1+\nu}{2} w_2 - 3\nu U_2\right) \sin\omega_2 t\right] \tag{103}$$

The absolute maximum of the stresses will then be: at the pole

$$\sigma_\phi = \sigma_\theta = \frac{pR_1}{2h_s} + \frac{E}{(1-\nu)\ R_1}\left[\left|w_o\right| + \left|w_2 - 3U_2\right|\right] \tag{104}$$

At the equator

$$\sigma_\phi = \frac{pR_1}{2h_s} + \frac{E}{(1=\nu^2)\ R_1}\left[(1+\nu)\ \left|w_o\right| + \left|\frac{1+\nu}{2} w_2 - 3U_2\right|\right]$$

$$\sigma_\theta = \frac{pR_1}{2h_s} + \frac{E}{(1-\nu^2)\ R_1}\left[(1+\nu)\ \left|w_o\right| + \left|\frac{1+\nu}{2} w_2 - 3\nu U_2\right|\right] \tag{105}$$

The coefficients W_o, U_2, W_2 are determined by evaluating the total momentum in the horizontal direction and the total kinetic energy of the balloon at the end of inflation. Thus:

$$\sum M_k \dot{x}_k = \int_o^n m_s R_1^2 \sin\phi\ (\dot{u}\cos\phi + \dot{w}\sin\phi)\ d\phi$$

$$\frac{1}{2} \sum M_k (\dot{x}_k^2 + \dot{y}_k^2) = \frac{1}{2} \int_o^n m_s R_1^2 \sin\phi\ (\dot{u}^2 + \dot{w}^2)\ d\phi \tag{106}$$

From which we obtain

$$\sum M_k \dot{x}_k = \frac{M}{8}\left[W_o \omega_o - \frac{W_2 + 6U_2}{8}\,\omega_2\right]$$

$$\sum M_k(\dot{x}_k^2 + \dot{y}_k^2) = \frac{M}{2n}\left[W_o^2\omega_o^2 + \frac{W_2^2 + 6U_2^2}{5}\,\omega_2^2\right] \qquad (107)$$

The coefficients U_2 and W_2 are related by Equation (93):

$$U_2 = \frac{2(1+\nu) - \Omega_2^2}{6(1+\nu)} \qquad (108)$$

VOLUME OF THE BALLOON DURING DEPLOYMENT

Referring to Figure 6, the volume of the kth fold will be given by:

$$V_k = \frac{a_o H_k^2}{6} \int_0^{\frac{\pi}{2}} \left[\cos \frac{s_{2k-1}}{R_1} + 2 \cos \frac{s_{2k-1} + H_k \alpha_1}{R_1} \right] d\alpha_1$$

$$+ \frac{a_o H_k}{2} \int_0^{s_{2k+1} - (s_{2k-1} + \frac{\pi}{2} H_k)} \left[\cos \frac{s_{2k+1} + \zeta}{R_1} \right.$$

$$\left. + \cos \frac{s_{2k-1} + \frac{\pi}{2} H_k + \zeta}{R_1} \right] d\zeta +$$

$$+ \frac{a_o H_k}{6} \int_{\frac{\pi}{2}}^{0} \left[\cos \frac{s_{2k+1}}{R_1} + 2 \cos \frac{s_{2k+1} - H_k \alpha_2}{R_1} \right] d\alpha_2$$

$$= \frac{a_o H_k R_1}{3} \left[\frac{\pi}{2} \frac{H_k}{R_1} \cos \frac{\ell_k}{R_1} + 5 \sin \frac{\ell_k}{R_1} + \right.$$

$$\left. + \sin \frac{\ell_k + \frac{\pi}{2} H_k}{R_1} \right] \cos \frac{s_k}{R_1}$$

where

$2\ell_k = s_{2k+1} - s_{2k-1}$ is the length of the kth fold.

$s_k = \frac{s_{2k+1} - s_{2k-1}}{2}$ is the distance along the surface of the balloon, from the center of the kth fold to the equator.

In practice, the length $2\ell_k$ and the height H_k of the accordion folds will be small as compared to R_1 the sphere radius. Hence we have:

$$V_k \sim 2 a_o H_k \ell_k \cos \frac{s_k}{R_1}$$

and the total volume will be

$$V = 2a_o \left[H_o \ell_o + 2 \sum_{1}^{\frac{N}{2}} H_k \ell_k \cos \frac{s_k}{R_1} \right] \qquad (109)$$

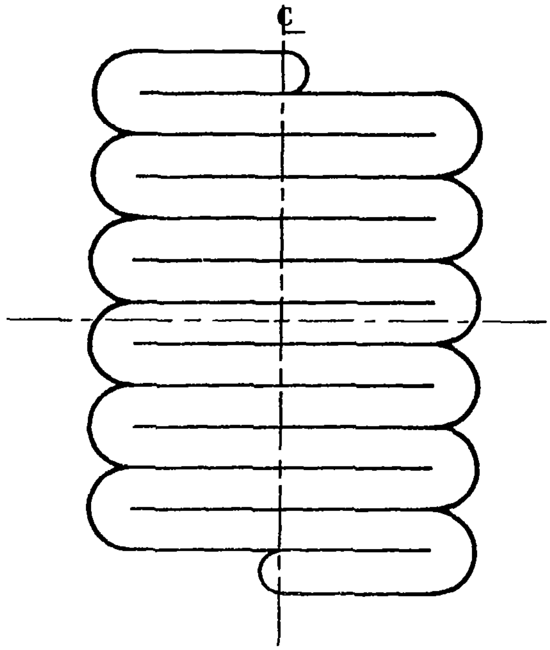


Figure 5 Side View of Balloon During Deployment (Schematic)

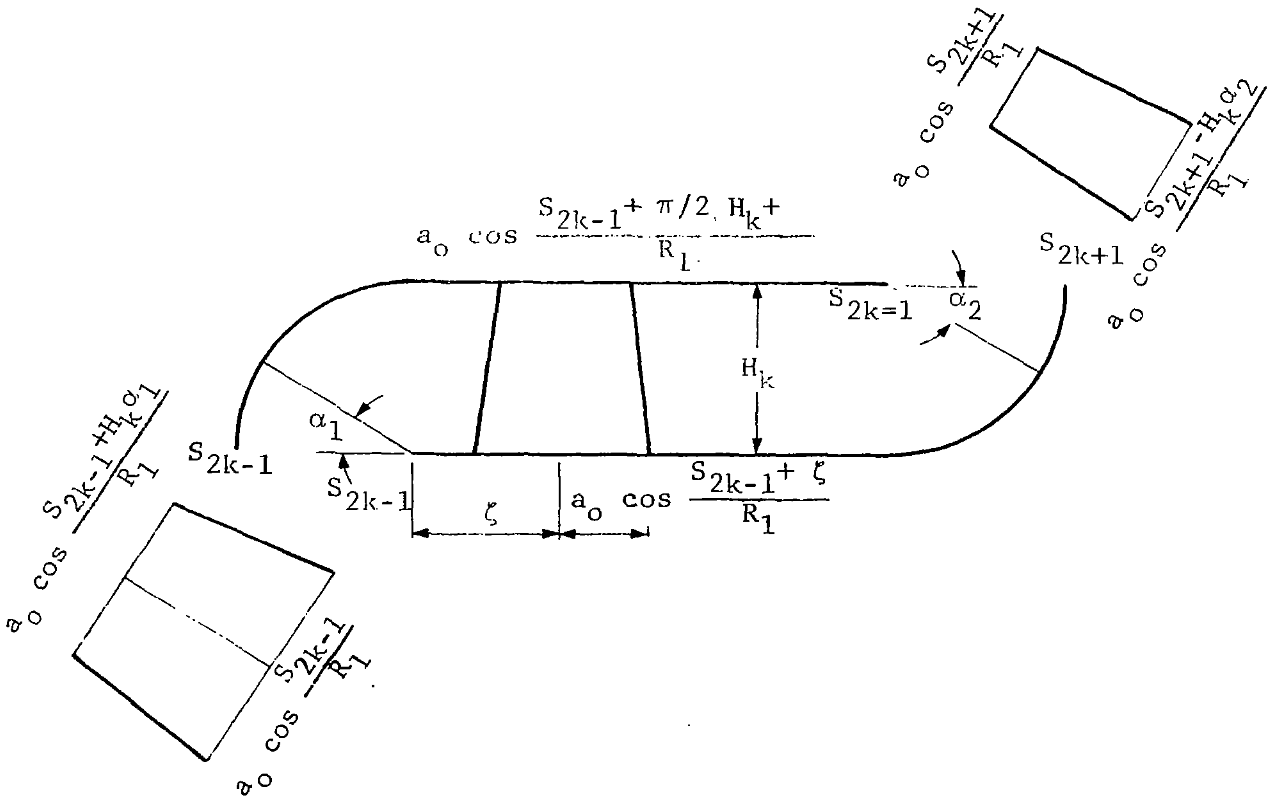


Figure 6 Side View of an Accordion Fold (Schematic)

Where N is the number of accordion folds. Assuming all the folds to be equal, i.e.,

$$H_o = H_1 = \cdots = H = \frac{2L}{N}$$

$$\ell_o = \ell_1 = \cdots = \ell = \frac{\pi}{2}\frac{R_1}{N}$$

then

$$s_k = 2k\ell = k\frac{\pi R_1}{N}$$

$$V = 2a_o H\ell\left[1 + 2\sum_{k=1}^{\frac{N}{2}} \cos k\frac{\pi}{N}\right] = 2a_o H\ell \cot\frac{\pi}{2N} \sim \frac{4a_o R_1 L}{N} \qquad (110)$$

The effective width a_o of the equatorial cross section is given by (Figure 7):

$$a_o = a_1 + \left[a_1 - \sqrt{a_1^2 - \left(\frac{2H_o}{n-4}\right)^2}\,\right]$$

where n is the number of pleat folds. For large n, we have approximately

$$a_o = a_1 = \frac{2\pi R_1}{n}$$

Finally:

$$V = \frac{8\pi R_1^2 L}{nN} = \pi a^2 L \qquad (111)$$

The radius a of the equivalent circular cross section at the equator is then:

$$a = \sqrt{\frac{8}{nN}}R_1$$

At the beginning of the inflation process, the length $2L_o$ of the accordion folded balloon is equal to the polar inside diameter of the canister, D_c and the initial volume V_o is

$$V_o = \frac{4\pi R_1^2 D_c}{nN}$$

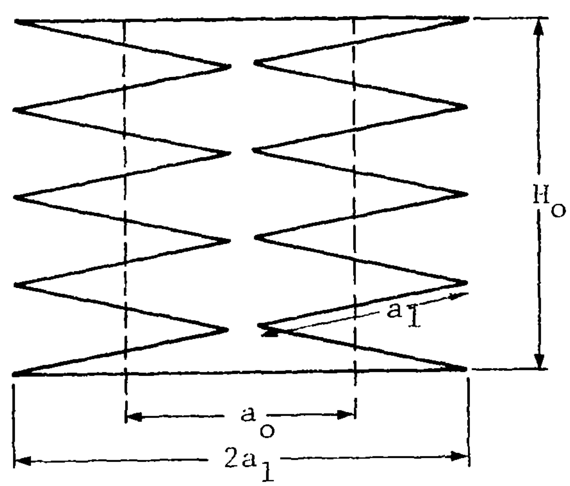


Figure 7 Cross Section of the Equatorial Fold (Schematic)

MODELING LAWS

Deployment Stage

This stage is governed by the differential Equation (26):

$$\frac{d^2L}{dt^2} = \frac{2\pi}{M}\left[F(L)\left(1 - \frac{L_o}{L}\right) + 2a^2\left[p_c + p_o\frac{L_o}{L}\right]\right] \tag{113}$$

where $F(L)$ is given by Equation (25).

For a geometrical similar model we have

$$\ell_m = \lambda \ell_p$$

where ℓ_p, ℓ_m are corresponding dimensions in the full-scale balloon and model and λ is the scale factor. The model will satisfy the differential equation

$$\frac{d^2L_m}{dt_m^{\ 2}} = \frac{2\pi}{M_m}\left[F_m(L_m)\left(1 - \frac{L_{om}}{L_m}\right) + 2a_m^{\ 2}\left[p_{cm} - p_{1m} + p_{om}\frac{L_{om}}{L_m}\right]\right] \tag{114}$$

where p_{1m} is the outside pressure acting on the model (test tank pressure).

Let

$$M_m = \mu M$$

$$t_m = \tau t$$

i.e., μ, τ, are the mass and time scale factors. Equation (114) can be written

$$\frac{\lambda}{\tau^2}\frac{d^2L}{dt^2} = \frac{2\pi}{\mu M}\left[F_m(\lambda L)\left(1-\frac{L_o}{L}\right) + 2\ \lambda^2 a\left[p_{cm} - p_{1m} + p_{om}\frac{L_o}{L}\right]\right] \tag{115}$$

or

$$\frac{d^2L}{dt^2} = \frac{\lambda\tau^2}{\mu}\ \frac{2\pi}{M}\left[\frac{F_m(\lambda L)}{\lambda^2}\left(1 - \frac{L_o}{L}\right) + 2a\left[p_{cm} - p_{1m} + p_{om}\frac{L_o}{L}\right]\right] \tag{116}$$

Comparing Equation (116) with Equation (113) we obtain the following modeling conditions

a) $\frac{\lambda\tau^2}{\mu} = 1$

b) $F_m(\lambda L) = \lambda^2 F(L)$

c) $p_{cm} - p_{1m} = p_c$

d) $p_{om} = p_o$

Hence

a) The time scale factor τ will be

$$\tau = \sqrt{\frac{\mu}{\lambda}}$$

b) The elasticity of the accordion folds must have the same functional form in model and full-scale balloon, and its scale factor must be equal to the square of the linear scale factor. Both conditions are satisfied if the model is folded in the same way as the balloon and the bearing force is proportional to the square of a characteristic length. As shown in Section III, the force $F(L)$ can be expressed approximately as

$$F(L) = \frac{\pi^3}{48} N^2 \, nER_1 h_s^3 \, \frac{f\frac{L}{L_1}}{L^2}$$

Then, condition (b) will be satisfied if

$$\frac{h_{sm}}{h_s} = \sqrt[3]{\frac{E}{E_m}} \, \lambda$$

If the model is made of the same material as the balloon, this implies that the thickness must be scaled also as the linear dimensions.

c) The pressure due to the subliming chemicals in the model must be equal to the pressure due to the subliming chemicals in the full-scale balloon plus the test tank pressure. This implies that, either we must use for the model test a chemical with a higher subliming pressure or carry the test at a higher temperature or both.

d) The residual gas pressure must be the same in model and and full-scale balloon.

Inflation Stage

The behavior of the balloon during this stage is governed by the differential Equations (39) and (40)

$$\begin{aligned} \ddot{x} &= \frac{1}{\bar{m}} \left[px \sin \phi - F_\theta + \frac{\partial}{\partial s} F_\phi \, x \cos \phi \right] \\ \ddot{y} &= \frac{1}{\bar{m}} \left[px \cos \phi - \frac{\partial}{\partial s} F_\phi \, x \sin \phi \right] \end{aligned} \tag{117}$$

where

$$\bar{m} = m_1 R_1 \sin \frac{s}{R_1} = \frac{M}{4\pi R_1} \sin \frac{s}{R_1}$$

again, if λ, μ, τ are the length, mass and time scale factors we have for the model

$$\begin{aligned} \frac{\lambda}{\tau^2} \ddot{x} &= \frac{\lambda}{\mu} \left[\lambda(p_m - p_{1m}) \, x \sin \phi - F_{\theta_m} + \frac{\partial}{\partial s} F_{\phi_m} \, x \cos \phi \right] \\ \frac{\lambda}{\tau^2} \ddot{y} &= \frac{\lambda}{\mu} \left[\lambda(p_m - p_{1m}) \, x \cos \phi - \frac{\partial}{\partial s} F_{\phi_m} \, x \sin \phi \right] \end{aligned} \tag{118}$$

comparing Equations (117) and (118) we obtain

a) $\dfrac{\lambda\tau^2}{\mu} = 1$ c) $F_{\phi m} = \lambda F_\phi$

b) $F_{\theta m} = \lambda F_\theta$ d) $p_m - p_{1m} = p$

Hence

a) The time scale factor is given by

$$\tau = \sqrt{\frac{\mu}{\lambda}}$$

b) The hoop force F_θ must be scaled as the linear dimensions. This condition will be satisfied if the hoop force is proportional to the parallel radius as for a linearly elastic body. In general, it will not be possible to satisfy this condition exactly (the thickness of the shell can not be scaled as the other dimensions). However, since this force will usually be small as compared with the others, its failure to satisfy exactly the scaling conditions will not seriously affect the results of the scale test.

c) The meridian force F_ϕ must scale as the linear dimensions. Solving the second of Equations (117) for F_ϕ we have

$$2\pi\ F_\phi\ x \sin\phi = \pi\ p\ x^2 - M(s)\ \ddot{y}_G \tag{119}$$

where $M(s)$ is the total mass of the shell above the point under consideration and y_G the ordinate of its centroid. For the model we have then:

$$2\pi\lambda F_{\phi_m}\ x \sin\phi = \pi\ \lambda^2 \left(p_m - p_{1m}\right) x^2 - \frac{\mu\lambda}{\tau^2} M(s)\ \ddot{y}_G \tag{120}$$

Finally, on account of (a) and (d)

$$F_{\phi_m} = \lambda\ F_\phi$$

d) The internal pressure in the model must be equal to the internal pressure in the full-scale balloon plus the pressure in the test tank, at all times, that is

$$p_{cm} - p_{1m} + p_{om}\ \frac{V_{1m}}{V_m} = p_c + p_o \frac{V_1}{V}$$

where

V_1 is the volume at the beginning of the inflation stage (end of deployment)

V is the present volume during inflation

p_c is the pressure due to sublimation of the chemicals

p_o is the pressure due to the residual gases at the beginning of the inflation stage (end of deployment).

The volume in model and full-scale balloon are related by:

$$V_m = \lambda^3 V$$

Hence, condition (d) requires

$$p_{cm} = p_c + p_{1m}$$

$$p_{om} = p_o$$

That is, the pressure due to the subliming chemicals in the model must be equal to the pressure due to the subliming compounds in the full-scale balloon plus the pressure in the test tank and the residual gas pressure must be the same for balloon and model.

Note on the Scaling Laws

In Appendix A it is shown, that during the deployment stage, heat is accumulated in the satellite at a rate larger than necessary to maintain a constant sublimation pressure while, during most of the inflation stage, the situation is reversed. This implies that, during deployment the pressure will follow the law:

$$p = p_c + p_o \frac{V_o}{V} \qquad (121)$$

with

p_c = pressure due to sublimation of the chemicals at launch temperature

p_o = pressure due to residual gases at launch

V_o, V = initial and present volume of the balloon during deployment while, during inflation, the pressure will be approximately

$$p = \bar{p} \frac{V_1}{V} \tag{122}$$

where

$\bar{p} = p_c + p_o \frac{V_o}{V_1}$ = total internal pressure at the end of the deployment stage (beginning of the inflation stage)

V_1 = volume of the balloon at the end of the deployment stage (beginning of the inflation stage)

V = present volume of the balloon during inflation

The scaling laws require that, for both stages

$$\begin{aligned} p_{cm} &= p_{1m} + p_c \\ p_{om} &= p_o \end{aligned} \tag{123}$$

By Equation (121) and (122), Equation (123) becomes

$$\begin{aligned} p_{cm} &= p_{1m} + p_c \\ p_{om} &= p_o \end{aligned} \tag{124}$$

for the deployment stage and

$$\begin{aligned} p_{cm} &= p_{1m} \\ p_{om} &= \bar{p} = p_c + p_o \frac{V_o}{V_1} \end{aligned} \tag{125}$$

for the inflation stage.

Hence, for correct modeling, it will be necessary to provide some source of heat so that the sublimation pressure p_{cm} in the model be maintained at the adequate constant value during each stage. Failure to satisfy this condition will not affect seriously the results at the beginning of the process as, in general, $p_c + p_{1m}$ will be small as compared to

$$p_o \frac{V_o}{V}$$

but, near the end of inflation, the pressure due to the residual gases has dropped to only a small fraction of its original value and hence p_{1m} will not be negligible as compared to

$$p_o \frac{V_o}{V} .$$

APPENDIX A

TEMPERATURE AND SUBLIMATION RATES

1. Temperature of the Satellite. - The temperature of the balloon satisfies the differential equation

$$M\ C_p \frac{dT}{dt} + \lambda_s \frac{dw}{dt} = \frac{dq_a}{dt} - \frac{dq_e}{dt} \tag{126}$$

where

$M = M_s + M_c$	is the total mass of the balloon (including the chemicals) (g_r)
C_p	is the specific heat of the whole balloon (including the chemicals) (erg gr^{-1} °K^{-1})
$\frac{dT}{dt}$	is the rate of change of the absolute temperature of the balloon (°K sec^{-1})
λ_s	is the latent heat of sublimation of the chemicals (erg gr^{-1})
$\frac{dw}{dt}$	is the rate at which the chemicals are sublimated (gr sec^{-1})
$\frac{dq_a}{dt}$	is the rate at which the balloon absorbs heat (erg sec^{-1})
$\frac{dq_e}{dt}$	is the rate at which the balloon emits heat (erg sec^{-1})

The rate at which the balloon absorbs heat from all sources (direct sun radiation, direct Earth radiation and reflected Earth radiation) is given by[1]

$$\frac{dq_a}{dt} = \left[1 + 2a_E(1 - \sqrt{1-k^2})(F_R(\overline{\beta}) + \frac{1-a_E}{2a_E} \frac{\alpha_E}{\alpha_s}\right] C_s \alpha_s S' \tag{127}$$

where

a_E is the Earth albedo (a_E = 0.36 approximately)

$k = \frac{R_o}{R_o+H_s}$

R_o is the Earth radius (R_o = 6.317 x 10^8 cm approximately)

Clemmons, D. L., Jr., The ECHO I Inflation System, NASA TN D-2194.

H_s is the altitude of the satellite orbit (cm)

$\bar{\beta}$ is the angle between the radius vector of the satellite and that of the Sun from the Earth center

$F_R(\bar{\beta})$ is the relative Earth reflected energy incident on the satellite which can be taken approximately as

$F_R(\bar{\beta}) = \cos\bar{\beta} \quad (0 \leq \bar{\beta} \leq \frac{\pi}{2})$

α_E is the absorptance of the satellite skin to Earth radiation

α_s is the absorptance of the satellite skin to solar radiation

C_s is the solar radiation constant ($C_s = 1.3953 \times 10^6$ erg cm^{-2} sec^{-1})

S' is the area of the meridian cross section of the satellite

The rate at which the balloon emits heat is given by

$$\frac{dq_e}{dt} = \varepsilon_o \, \sigma \, ST^4 \tag{128}$$

where

ε_o is the total emittance coefficient of the satellite skin

σ is the Stephan-Boltzman constant ($\sigma = 5.71 \times 10^{-5}$ erg cm^{-2} sec^{-1} °K^{-4})

S is the surface area of the satellite

T is the absolute temperature of the satellite

By (127) and (128) Equation (126) can be written

$$M \, C_p \, \frac{dT}{dt} + \lambda_s \, \frac{dw}{dt} = \varepsilon_o \, \sigma \, S(T_1{}^4 - T^4) \tag{129}$$

where

$$T_1 = \sqrt[4]{\left[1 + 2a_E(1 - \sqrt{1-k^2})(F_R(\bar{\beta}) + \frac{1-a_E}{2a_E}\frac{\alpha_E}{\alpha_s}\right] \frac{C_s\alpha_s}{\varepsilon_o\sigma} \, \frac{S'}{S}} \tag{130}$$

If in Equation (129) we set $\lambda_s = 0$, i.e. there are no chemical substances to be sublimated or, in other words, all the heat accumulated in the balloon is used to raise the temperature so the skin will heat up at the highest possible rate, we obtain

$$MC_p \frac{dT}{dt} = \varepsilon_o \sigma s \,(T_1^4 - T^4) \tag{131}$$

The sample calculations below have been carried out for the PAGEOS satellite with the following data.*

<u>Data</u>

Sphere radius	$R_1 = 50 \text{ ft} = 1524 \text{ cm}$
Skin thickness	$h_s = 5 \times 10^{-4} \text{ in.} = 1.27 \times 10^{-3} \text{ cm}$
Skin density	$\rho_s = 1.38 \text{ gr cm}^{-3}$
Skin mass	$M_s = 4\pi R_1^2 h_s \rho_s = 5.115 \times 10^4 \text{ gr}$
Modulus of Elasticity	$E = 6.6 \times 10^5 \text{ lb in.}^{-2}$
	$= 4.55 \times 10^{10} \text{ dy cm}^{-2}$
Number of pleat folds	$n_1 = 418$
Initial equatorial radius	$a = 2\pi R_1/n_1 = 22.91 \text{ cm}$
Initial length	$2L_o = \pi a = 71.97 \text{ cm}$
Deployment length	$2L_1 = \pi R_1 = 4.788 \times 10^3 \text{ cm}$
Altitude of orbit	$H_s = 822 \text{ n.m.} = 1.523 \times 10^8 \text{ cm}$
Solar absorptance	$\alpha_s = 0.10$
Earth absorptance	$\alpha_E = 0.03$
Thermal emittance	$\varepsilon_o = 0.03$
Launch temperature	$T_o = 75°F = 297°K$
Specific heat	$C_p = 1.3 \times 10^7 \text{ erg gr}^{-1} \,°K^{-1}$

Then

$$k = \frac{6.371}{6.371 + 1.523} = 0.808$$

$$1 - \sqrt{1-k^2} = 0.411$$

---

*See note page 59

Taking the maximum value for $F_R(\bar{\beta})$, $F_R(\bar{\beta}) = 1$, i.e., assuming that the satellite is on the Earth-Sun line, we have

$$T_1{}^4 = \left[1 + 2x.36x.411\ (1 + \frac{1-.36}{4x.36}\ \frac{0.03}{0.10})\right]\ \frac{1.3953x10^6x.10}{0.03x5.71x10^{-5}}\ \frac{S'}{S}$$

$$= 1.099x10^{11}\ \frac{S'}{S}$$

$$\frac{\varepsilon_o \sigma}{MC_p} = \frac{0.03x5.71x10^{-5}}{5.115x10^4x1.3x10^7} = 2.576x10^{-18}$$

a) Beginning of deployment

$$S = 4\ a\ L_o\ \left[\sqrt{2} + \log_e(1+\sqrt{2})\right] = 7.694x10^3\ cm^2$$

$$S' = \frac{8\ a\ L_o}{\pi} = 2.136x10^3\ cm^2$$

$$T_1{}^4 = 1.099x10^{11}x\ \frac{2.136}{7.694} = 3.051x10^{10}$$

$$T_o{}^4 = 297^4 = 7.781x10^9$$

$$\frac{dT}{dt} = 2.576x10^{-18}\ \left[3.051x10^{10} - 7.781x10^9\right]\ 7.69x10^3$$

$$= 4.505x10^{-4}\ {}^\circ K/sec$$

b) End of deployment (beginning of inflation)

$$S = 4\ a\ L_1 \left[\sqrt{1 + \left(\frac{L_o}{L_1}\right)^2} + \frac{\log_e\left(\frac{L_o}{L_1} + \sqrt{1 + \left(\frac{L_o}{L_1}\right)^2}\right)}{\frac{L_o}{L_1}}\right]$$

$$= 4.474x10^5\ cm^2$$

$$S' = \frac{8\ a\ L_1}{\pi} = 1.425x10^5\ cm^2$$

$$T_1{}^4 = 1.099x10^{11}\ x\ \frac{1.425}{4.474} = 3.499x10^{10}$$

Assuming $T = T_o$ we get

$$\frac{dT}{dt} = 2.576 \times 10^{-18} \left[3.499 \times 10^{10} - 7.781 \times 10^{9}\right] 4.474 \times 10^{5}$$
$$= 3.137 \times 10^{-2} \, °K/sec$$

c) at the end of inflation

$$S = 4\pi R_1^{\,2} = 2.919 \times 10^{7} \text{ cm}^2$$

$$S' = \pi R_1^{\,2} = 7.297 \times 10^{6} \text{ cm}^2$$

$$T_1^{\,4} = 1.099 \times 10^{11} \; \frac{0.7297}{2.919} = 2.747 \times 10^{10}$$

assuming again $T = T_o$ we get

$$\frac{dT}{dt} = 2.576 \times 10^{-18} \left[2.747 \times 10^{10} - 7.781 \times 10^{9}\right] 2.919 \times 10^{7}$$
$$= 1.480 \, °K/sec$$

As the present temperature T is larger than T_o, the temperature when the canister opens, the actual rates of change of temperature will be smaller than the above computed values. Moreover, in the above calculation we did not consider the subliming chemicals which will increase the mass M and the specific heat C_p and consequently reduce the rate of change of temperature with time. Moreover, the sublimation of the chemical powder will require a certain amount of heat which will further reduce the value of dT/dt.

2. Rate of Sublimation of the Chemical Compounds. - If in Equation (129) we set $dT/dt = 0$, we obtain

$$\lambda_s \; \frac{dw}{dt} = \varepsilon_o \, \sigma \, S \, (T_1^{\,4} - T^4) \tag{132}$$

i.e., all the heat accumulated in the satellite is used to sublimate the chemicals or, in other words, the chemicals will be sublimated at the highest possible rate.

Let p_c, V be the pressure due to the sublimated chemicals and the volume of the balloon at time t. Then, assuming perfect gas,

$$p_c V = \frac{R_g}{M_o} T w \tag{133}$$

where

R_g is the universal gas constant (8.3149×10^7 erg·Mol^{-1}·°K^{-1})
M_o is the molecular weight of the compound (gr·Mol^{-1})

Assuming constant temperature, we have that, in order to keep a constant pressure, the rate of change of volume must be

$$\frac{dV}{dt} = \frac{dV^*}{dt} = \frac{R_g T}{M_o p_c} \frac{dw}{dt} = \frac{R_g T \varepsilon_o \sigma}{M_o \lambda_s p_c} (T_1^4 - T^4) S \tag{134}$$

If the actual rate of change of volume is larger than the above value, there will be a drop in the sublimation pressure.

The sample calculations below have been carried out for the PAGEOS satellite using benzoic acid as the subliming compound and the following data.

Temperature	$T = 300°K$
Molecular weight	$M_o = 122.12$ gr Mol^{-1} (from Table 1)
Latent heat of sublimation	$\lambda_s = 5.60 \times 10^9$ erg gr^{-1} (from Table 1)
Sublimation pressure	$p_c = e^{29.595 - \frac{8223}{300}} = 8.89$ dy cm^{-2}

The other data are the same as in Section 1 of this Appendix. Hence

$$\frac{R_g T \varepsilon_o \sigma}{M_o \lambda_s p_c} = \frac{8.3149 \times 10^7 \times 300 \times .03 \times 5.71 \times 10^{-5}}{122.12 \times 5.60 \times 10^9 \times 8.89}$$

$$= 7.028 \times 10^{-9} \text{ cm sec}^{-1} {}^\circ K^{-4}$$

$$T^4 = 8.1 \times 10^9 {}^\circ K^4$$

TABLE I

PHYSICAL PROPERTIES OF SOME ORGANIC COMPOUNDS

Compound	Formula	Molecular Weight gr/Mol	$\log_{10} p(\text{torr}) = a - \frac{b}{T}$		$\log_e p(\text{dy cm}^{-2}) = A - \frac{B}{T}$		Latent heat of Sublimation Kcal/Mol	Latent heat of Sublimation 10^9 erg/gr
1. Acetamide	C_2H_5NO	59.07	9.09	3066	28.134	7060	14.0	9.94
2. Benzoic Acid	$C_7H_6O_2$	122.12	9.73	3571	29.595	8223	16.3	5.60
3. Naphtalene	$C_{10}H_8$	128.16	10.75	3616	31.949	8326	16.5	5.40
4. d-Camphor	$C_{10}H_{16}O$	152.23	8.41	2645	26.571	6090	12.1	3.33
5. Anthra-quinone	$C_{14}H_8O_2$	208.20	14.31	6604	40.146	15206	30.2	6.07
6. Anthracene	$C_{14}H_{10}$	178.22	11.15	5401	32.870	12436	24.7	5.80
			a	b	A	B		

a) Beginning of deployment

$$S = 7.694 \times 10^3 \text{ cm}^2$$

$$S' = 2.136 \times 10^3 \text{ cm}^2$$

$$T_1^{\,4} = 3.051 \times 10^{10} {}^\circ K^4$$

$$\frac{dV^*}{dt} = 7.028 \times 10^{-9} \left[3.051 \times 10^{10} - 8.1 \times 10^9\right] 7.694 \times 10^3$$

$$= 1.212 \times 10^6 \text{ cm}^3 \text{ sec}^{-1}$$

while the rate of change of volume due to inflation is

$$\frac{dV}{dt} = 0$$

b) End of deployment

$$S = 4.474 \times 10^5 \text{ cm}^2$$

$$S' = 1.425 \times 10^5 \text{ cm}^2$$

$$T_1^{\,4} = 3.499 \times 10^{10}$$

$$\frac{dV^*}{dt} = 7.028 \times 10^{-9} \left[3.499 \times 10^{10} - 8.1 \times 10^9\right] \times 4.474 \times 10^5$$

$$= 8.455 \times 10^7 \text{ cm}^3/\text{sec}$$

while the actual rate of change of volume is

$$\frac{dV}{dt} = 7.731 \times 10^6 \text{ cm}^3/\text{sec}.$$

c) Beginning of inflation. The shape and size of the balloon are the same as for the end of deployment, hence

$$\frac{dV^*}{dt} = 8.455 \times 10^7 \text{ cm}^3/\text{sec}$$

while the actual rate of change of volume is

$$\frac{dV}{dt} = 4.729 \times 10^6 \text{ cm}^3/\text{sec}$$

d) At 5 percent inflation (Equatorial diameter = 0.05 x Final diameter).

$$S' = 0.050 \times 1.566 \times 7.297 \times 10^6 = 5.714 \times 10^5 \ cm^2$$

$$S = \pi S' = \pi \times 5.714 \times 10^5 = 1.795 \times 10^6 \ cm^2$$

$$T_1^{\ 4} = 1.099 \times 10^{11} \times \frac{0.5714}{1.795} = 3.498 \times 10^{10} \ {}^\circ K^4$$

$$\frac{dV^*}{dt} = 7.028 \times 10^{-9} \left[3.498 \times 10^{10} - 8.1 \times 10^9 \right] 1.795 \times 10^6$$

$$= 3.391 \times 10^8 \ cm^3/sec$$

while the actual rate of change of volume is

$$\frac{dV}{dt} = 3.426 \times 10^8 \ cm^3/sec$$

e) At the end of inflation

$$S' = \pi R_1^{\ 2} = 7.297 \times 10^6 \ cm^2$$

$$S = 4\pi R_1^{\ 2} = 2.919 \times 10^7 \ cm^2$$

$$T_1^{\ 4} = 1.099 \times 10^{11} \ \frac{0.7297}{2.919} = 2.747 \times 10^{10} \ {}^\circ K^4$$

$$\frac{dV^*}{dt} = 7.028 \times 10^{-9} \left[2.747 \times 10^{10} - 8.1 \times 10^9 \right] 2.919 \times 10^7$$

$$= 3.974 \times 10^9 \ cm^3/sec$$

while the actual rate of change of volume is

$$\frac{dV}{dt} = 2.074 \times 10^{10} \ cm^3/sec$$

The values of $\frac{dV}{dt}$ above have been calculated on the assumption that the pressure due to the sublimation of the chemicals remained constant during the whole process.

It can be seen, that during the deployment stage, the rate at which heat is accumulated in the satellite is larger than what is required to maintain the sublimation gas pressure. On the other hand, the rate of change of temperature during this stage is certainly less than the calculated values for the skin alone which were at most a few hundredths of a degree per second.

It seems reasonable then to assume, that during the few seconds it takes the balloon to reach full deployment, the pressure due to the sublimation of the chemicals remains constant and the one due to the residual gases varies according to Boyle's law.

During most of the inflation stage the rate of heat accumulation is smaller than the amount required to just keep the sublimation pressure constant. To calculate the actual pressure inside the balloon, we determine the rate of sublimation of the chemical from Equation (132)

$$\frac{dw}{dt} = \frac{\varepsilon_o \sigma S}{\lambda_s} (T_1^4 - T^4) \tag{110'}$$

Assuming that the rate of of sublimation remains constant during a short time interval Δt, the amount of gases generated during said time interval will be:

$$\Delta w = \frac{\varepsilon_o \sigma S}{\lambda_s} (T_1^4 - T^4)\Delta t \tag{135}$$

Let p_n, V_n, be the pressure and volume of the balloon at time t_n. At time $t_{n+1} = t_n + \Delta t$ the partial pressure due to the gases already in the balloon at $t = t_n$ will be

$$p'_{n+1} = p_n \frac{V_n}{V_{n+1}} \tag{136a}$$

While the gases generated during the time interval Δt will give a partial pressure (Equation (133):

$$p''_{n+1} = \frac{R_g T}{M_o} \frac{\Delta w}{V_{n+1}} \tag{136b}$$

Hence the total pressure at $t = t_{n+1}$ will be:

$$p_{n+1} = p'_{n+1} + p''_{n+1} = p_n \frac{V_n}{V_{n+1}} + \frac{R_g T}{M_o} \frac{\varepsilon_o \sigma S}{\lambda_s} \frac{T_1^4 - T^4}{V_{n+1}} \Delta t \tag{137}$$

Note. - After this Appendix was submitted, we received information that the actual altitude of the PAGEOS orbit will be 4250 Km and the actual temperature close to 140°F instead of 1523 Km and 75°F as used in the calculations. The higher altitude will have the effect of reducing the amount of heat received from the earth and hence will reduce the value of T_1, while the higher temperature will increase the value of T and p_c. Hence, the rate of change of temperature dT/dt and the rate of sublimation of the chemicals dw/dt, hence dV*/dt, will be smaller while the actual rate of change of volume dV/dt will be larger than the calculated values. Consequently, the assumption that the temperature remains constant will be even more valid.

APPENDIX B

COMPUTER PROGRAMS AND NUMERICAL EXAMPLES

Computer Program for the First Stage Deployment Time

1. Purpose.- The purpose of this program is to calculate the time for the first stage deployment by evaluation of the following integral (Equation (20)).

$$t = \int_{\lambda=L_o}^{\lambda=L} \frac{d\lambda}{\sqrt{GI(\lambda) + CI(\lambda)}} = \int_{\lambda=L_o}^{\lambda=L} \frac{d\lambda}{(\dot{L})} \tag{138}$$

λ ... dummy variable for L

$$(\dot{L}) = \sqrt{GI(\lambda) + CI(\lambda)}, \text{ where } CI(\lambda) = \int_{\alpha=L_o}^{\alpha=\lambda} CII(\alpha)\, d\alpha \tag{139}$$

$GI(\lambda)$ ... explicit function of λ (i.e. the part of $(\dot{L})$ which can be integrated explicitly from $\ddot{L}$)

$CI(\lambda)$ ... integral function of λ

$CII(\alpha)$... the part of $\ddot{L}$ that cannot be integrated explicitly

The particular form of $(\dot{L})$ used in this study, given by Equation (28) is an explicit function of λ hence, $CI(\lambda) \equiv 0$.

If at some future time a different formulation for $(\dot{L})$ is developed which cannot be entirely expressed as an explicit function of λ, (i.e. $CI(\lambda) \neq 0$) the program has the option to handle this case readily.

2. Input.- The input consists of:

Balloon parameters:

(a) Modulus of elasticity of the balloon skin. (E)

(b) Thickness of the skin. (HS)

(c) Inside diameter of the canister. (DC)

(d) Total weight of the balloon including the chemicals. (W)

(e) Diameter of the fully inflated balloon. (D1)

(f) Number of accordion folds. (NAF)

(g) Number of meridian pleat folds. (NPF)

Pressure parameters:

(a) Pressure of the residual gases inside the balloon at the beginning of the deployment stage. (POR) This value can usually be set equal to the residual gas pressure in the canister.

(b) Sublimation pressure of any chemicals present inside the balloon. (PCC) In view of the discussion in Appendix A, this value is assumed to remain constant during the entire deployment stage.

Output controls:

(a) Number of intermediate time print-outs for one case (including the final time print-out for the first stage). (JJ)

(b) A set of percent deployment values corresponding to each value of JJ in 3(a) above. (CL)

For example, if four intermediate time print-outs of the first stage are desired, JJ=4, CL=0.125, 0.25, 0.5, 1.0. Note: the decimal equivalent of the percent is read in as input (e.g. 50 percent is read in as 0.5). The range of input values should not exceed the following inequalities;

$$\frac{2}{\pi} \cdot \frac{DC}{12 \cdot DI} \cdot (1 + GAM) < CL \leq 1,$$

where the lower value corresponds to the beginning of deployment just beyond the singularity.*

---

* See following section (Integration Controls) for description of this singularity.

Integration controls:

The following four control variables (GAM, N, IS, NIM) are for controlling the integration loops of the first stage. Suggested values for the control values are:

GAM = 0.01
N = 30
IS = 0 (If CI(λ) = 0, IS must be zero)
NIM = 30

Each of these four variables is described below.

The integral, Equation (135), evaluated by this program has a singularity of the first kind at $L = L_o$. In the neighborhood of the singularity (i.e. from L_o to $L_o + \varepsilon$) it has been evaluated analytically, hence the integration routine begins after the singularity (i.e. at $L = L_o + \varepsilon$). The computer variable GAM is related to ε by the relation $\varepsilon = \text{GAM}\cdot L_o$.

The outer integration index N is related to the total number of integration intervals by the following relation: 6·N·JJ = total number of integration intervals for one case.

The inner integration switch is directly related to CI(λ). The meaning of the expression CI(λ) (given by Equation (136) has already been explained. If CI(λ) = 0, then set IS = 0; if at some future time a different expression for $\dot{L}$ is derived such that CI(λ) $\neq$ 0, then set IS = 1, and rewrite the ACC, CII and GI cards in the Fortran deck accordingly.

The variable NIM relates to the integration interval of the inner integral given by Equation (136). The total number of integration intervals in Equation (136) (at $\alpha = L$) is given by $6\cdot\left[\text{NIM} + 2\right]$. If IS = 0, NIM is not used by the program; if IS $\neq$ 0 a suggested value for NIM is 30.

SUMMARY OF INPUT CARDS FOR FIRST STAGE

Text Variable	Computer Variable	Definition and Units of Input	Format
p_o	POR	Residual Gas Pressure (torr)	E 10.5
p_c	PCC	Sublimation Pressure (torr)	E 10.5
E	E	Modulus of Elasticity (psi)	E 10.5
D_c	DC	Inside Diameter of Canister (in.)	E 10.5
h_s	HS	Skin Thickness (mils)	E 10.5
W	W	Total Weight of Balloon (including chemicals)(lbs)	E 10.5
D_1	D1	Diameter of Inflated Balloon (ft)	E 10.5
T	TP		E 10.5
N	NAF	Number of Accordion Folds	I 5
n_1	NPF	Number of Meridian Pleat Folds	I 5
	JJ	Number of Time Printouts	I 5
T	TP	Average Temperature of Balloon(°F)	E 10.5
		END OF CARD	
	N	Outer Integration index	I 5
	IS	Inner Integration Switch	I 5
	NIM	Inner Integration Index	I 5
	GAM	Singularity Percentage Control	E 10.5
		END OF CARD	
L/L_1	CL	Percent Deployment (8 per card)	8 E 10.5
		END OF $\left[\text{integer part of } (1 + \frac{JJ}{8})\right]$ CARDS	

There are, therefore $3 + \left[\text{integer part of } (1 + \frac{JJ}{8})\right]$ cards of required input for operation of the first stage program.

3. Output.- The first part of the output includes the input which is then readily available for checking and reference purposes. The ratio of the spring force of the elastic folds to the initial pressure force is printed (FBI) which gives a measure of the relative strength of these two acting forces. Also the ratio of the initial residual gas pressure to the chemical gas pressure (PB) is printed.

Finally, JJ groups of the following five quantities are printed:

PRESSURE	Current internal pressure inside balloon (torr)
ACC	Current acceleration at tip of balloon (ft/sec/sec)
STRESS	Current hoop stress at the equator (psi)
VELOCITY	Current velocity at tip of balloon (ft/sec)
TIME	Current time in seconds
LENGTH	Current value of L in feet (L is one-half the balloon length during deployment).

4. Cutoff.- The program arrives at a normal exit and calls for a new problem. At the end of the last problem the program stops.

5. Method.- The integration method used was "Weddle's rule".[2] Briefly the method breaks the integrand into groups of six intervals. The typical six interval region is given as:

$$\int_{x=D}^{x=D+6h} F(x)\,dx = \frac{3\cdot h}{10}\Big[f(D) + 5\cdot f(D+h) + f(D+2\cdot h) + 6\cdot f(D+3\cdot h) + f(D+4\cdot h) + 5\cdot f(D+5\cdot h) + f(D+6\cdot h)\Big]$$

The error over this interval is less than $\left|\frac{h\cdot\delta^6}{140}\right|$ where δ^6 is the sixth difference, as compared to say the less accurate Simpson's rule where the error is less than $\left|\frac{h\cdot\delta^4}{90}\right|$, where δ^4 is the fourth difference.

---

[2] John Todd, "Survey of Numerical Analysis," Mc-Graw Hill Book Co., 1962, p. 61.

The basic six interval formula is given, however, by combining like terms it can be written as a double summation over a $6\cdot h\cdot N$ span (where N is any integer ≥ 2).

$$\int_{x=D}^{x=D+6\cdot h\cdot N} F(x)\,dx = \frac{3\cdot h}{10} \sum_{m=1}^{m=N} \sum_{\ell=1}^{\ell=7} G_\ell \cdot Q_{m\ell} \cdot f(D + h\cdot\left[m\cdot 6 - \ell + T_{m\ell}\right])$$

for $n \geq 2$

where

$$G_\ell = \begin{pmatrix} 5 \\ 1 \\ 6 \\ 1 \\ 5 \\ 2 \\ 1 \end{pmatrix} \qquad \begin{aligned} T_{m\ell} &= 6\cdot N + 1 & & m = 1,\ \ell = 7 \\ &= 0 & & \text{all other } m,n \end{aligned}$$

$$\begin{aligned} Q_{m\ell} &= 1/2 & & m = 1,\ \ell = 6 \\ &= 0 & & m = 2,3,\ldots N,\ \ell = 7 \\ &= 1 & & \text{all other } m,\ell \end{aligned}$$

With reference to Equation (135)

$f(x)$ corresponds to $\dfrac{1}{(\dot{L})} = \dfrac{1}{\sqrt{GI + CI}}$,

D corresponds to $L_o + \varepsilon$

$D + 6\cdot h\cdot N$ corresponds to L

where ε is a small distance from the singularity which exist at L_o.

The value of the integral in the neighborhood of the singularity is calculated in the following way:

$$t_\varepsilon = \int_{L_o}^{\lambda=L_o+\varepsilon} \frac{d\lambda}{\sqrt{Q(\lambda) - Q(L_o)}},$$

where

$$Q(\lambda) - Q(L_o) = \int_{L_o}^{x=\lambda} \frac{4\pi}{M}\left[F(x)\cdot\left(1 - \frac{L_o}{x}\right) + 2\cdot a^2\cdot p(x)\right] dx$$

Let

$$\lambda = L_o + \gamma$$

$$d\lambda = d\gamma$$

Then

$$t_\varepsilon = \int_0^{\gamma=\varepsilon} \frac{d\gamma}{\sqrt{Q(L_o + \gamma) - Q(L_o)}}$$

Expanding Q in a Taylor series:

$$Q(L_o + \gamma) = Q(L_o) + \gamma Q'(L_o) + \frac{\gamma^2}{2} Q''(L_o) + \ldots$$

where the primes denote differentiation with respect to the argument. Neglecting the second and higher powers of γ we obtain:

$$t_\varepsilon \doteq \int_0^\varepsilon \frac{d\gamma}{\sqrt{Q'(L_o)\gamma}} = \frac{1}{\sqrt{Q'(L_o)}} \int_0^\varepsilon \frac{d\gamma}{\sqrt{\gamma}} = \frac{2\sqrt{\varepsilon}}{\sqrt{Q'(L_o)}} = \frac{\sqrt{\varepsilon}}{a\sqrt{\frac{\pi \cdot 2}{M}(p_o + p_c)}}$$

where

$$Q'(L_o) = \frac{8\pi \cdot a^2}{M}\left[p_o + p_c\right]$$

The t_ε value is added to the time obtained by the numerical integration.

6. <u>Sample problem</u>.- The PAGEOS balloon will be used as a sample problem.

Parameter	Computer Variable	Value
Residual Gas Pressure	POR	1.0 torr
Sublimation Pressure	PCC	0.01778 torr
Modulus of Elasticity	E	0.66×10^{6} psi
Inside Diameter of Canister	DC	26.5 in.
Skin Thickness	HS	0.5 mils
Total Weight of Balloon	W	147.5 lb
Diameter of Inflated Balloon	D1	100 ft
Temperature of Balloon	TP	100°F
Number of Accordion Folds	NAF	85
Number of Meridian Pleat Folds	NPF	418
Number of Time Printouts	JJ	4
Outer Integration Index	N	30
Inner Integration Switch	IS	0
* Inner Integration Index	NIM	30
Singularity Percentage Control	GAM	0.01
Percent Deployment	CL	0.125, 0.25, 0.5, 1.0

Result: Time for first stage deployment equals 6.98 sec.

The printout sheet for this sample calculation can be found on the following page.

* This value is not used by the program whenever IS = 0, therefore it is immaterial what value is inserted for NIM.

```
POR=  0.10000000E 01   PCC=  0.17780000E-01   E=  0.66000000E 06   DC=  0.24500000E 02   HS=  0.50000000E 00

W=  0.14750000E 03   D1=  0.10000000E 03   TP=  0.10000000E 03   NAF=   85   NPF=  418   JJ=    4

N=   30   IS=    0   NIM=   30   GAM=  0.10000000E-01

FB1=  0.22268577E 01   PB=  0.17780000E-01



PRESSURE=  0.13024449E 00    (TORR)      STRESS=  0.56720347E 01    (PSI)

ACC=  0.30855492E 01    (FT/SEC.SQ.)
VELOCITY=  0.14756803E 02    (FT/SEC)
TIME=  0.98929700E 00   (SEC)     LENGTH=  0.98179133E 01   (FT)



PRESSURE=  0.74012246E-01    (TORR)      STRESS=  0.64463382E 01    (PSI)

ACC=  0.98958355E 00    (FT/SEC.SQ.)
VELOCITY=  0.15856432E 02    (FT/SEC)
TIME=  0.16264091E 01   (SEC)     LENGTH=  0.19635827E 02   (FT)



PRESSURE=  0.45896123E-01    (TORR)      STRESS=  0.79949453E 01    (PSI)

ACC=  0.39443395E 00    (FT/SEC.SQ.)
VELOCITY=  0.16567252E 02    (FT/SEC)
TIME=  0.28334027E 01   (SEC)     LENGTH=  0.39271653E 02   (FT)



PRESSURE=  0.31838061E-01    (TORR)      STRESS=  0.11092160E 02    (PSI)

ACC=  0.28375219E 00    (FT/SEC.SQ.)
VELOCITY=  0.17329612E 02    (FT/SEC)
TIME=  0.51477191E 01   (SEC)     LENGTH=  0.78543306E 02   (FT)
```

7. Block diagram variables.-

t_ε ... time accumulated in region of singularity

VEL ... $\dot{L}$ (velocity at tip of balloon)

$$\int_{\lambda=CL_{i-1}}^{\lambda=CL_i} \frac{d\lambda}{\sqrt{GI(\lambda) + CI(\lambda)}} \sim QUAD = \frac{3\cdot H}{10} \sum_{m=1}^{N} \sum_{\ell=1}^{7} G_\ell \cdot Q_{m\ell} \cdot \left[\frac{1}{\sqrt{GI(TAU)+CI(TAU)}} \right]$$

where

$$TAU = CL_{i-1} + H\cdot(m\cdot 6 - \ell + T_{m\ell})$$

$$H = \frac{CL_i - CL_{i-1}}{6\cdot N}$$

CL corresponds to the text variable L

$$CI(TAU) = \int_{\alpha=L_o}^{\alpha=\lambda} CII(\alpha)d\alpha \sim QQUAD = \frac{3\cdot HH}{10} \sum_{p=1}^{NN} \sum_{q=1}^{7} G_q \cdot QQ_{pq} \cdot CII(ALP)$$

where

$$ALP = CLO + HH\cdot(p\cdot 6) - q + TT_{pq}$$

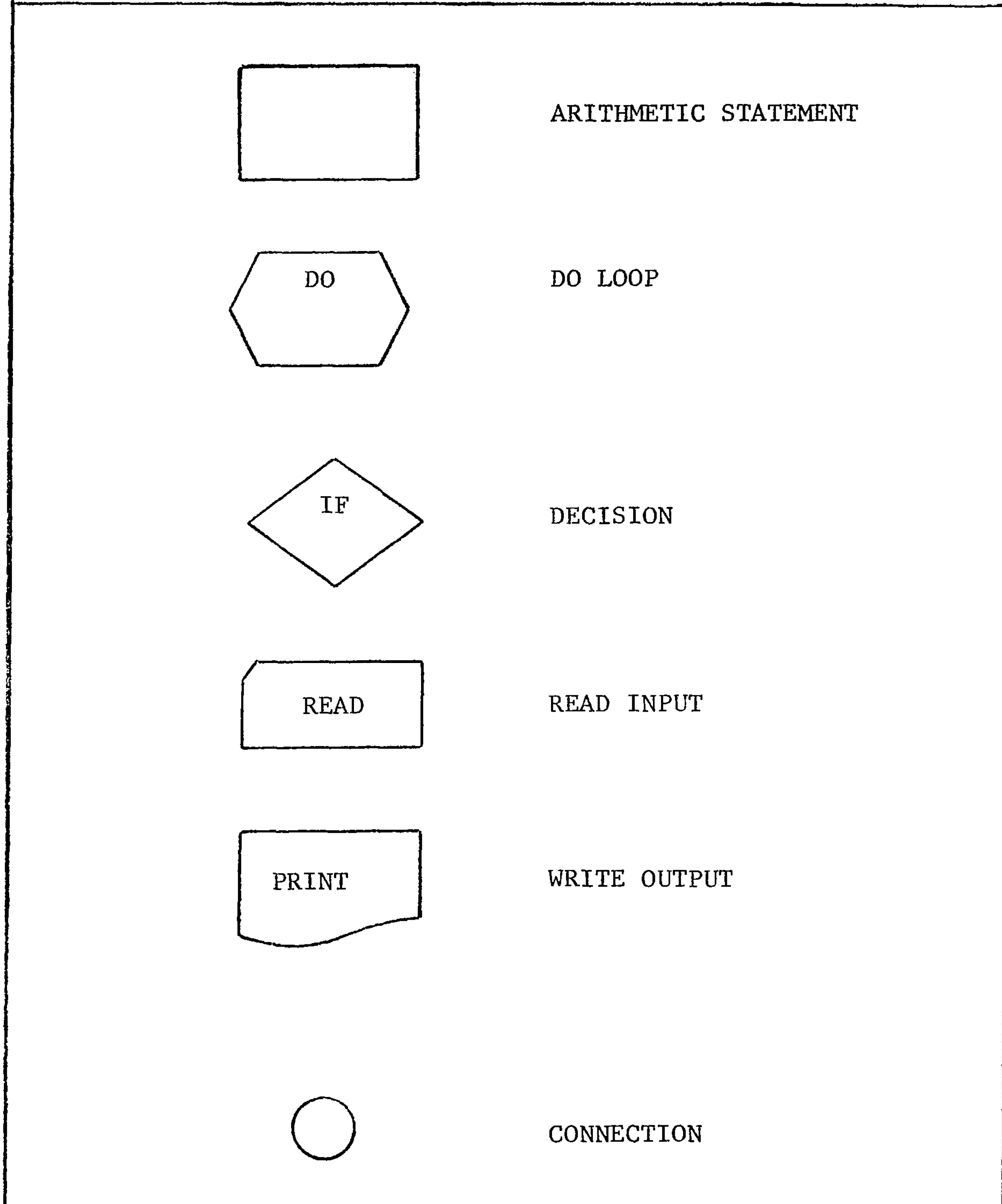
$$HH = \frac{TAU - CLO}{6\cdot NN}$$

QQ_{pq}, TT_{pq} have the same meaning as $Q_{m\ell}$, $T_{m\ell}$

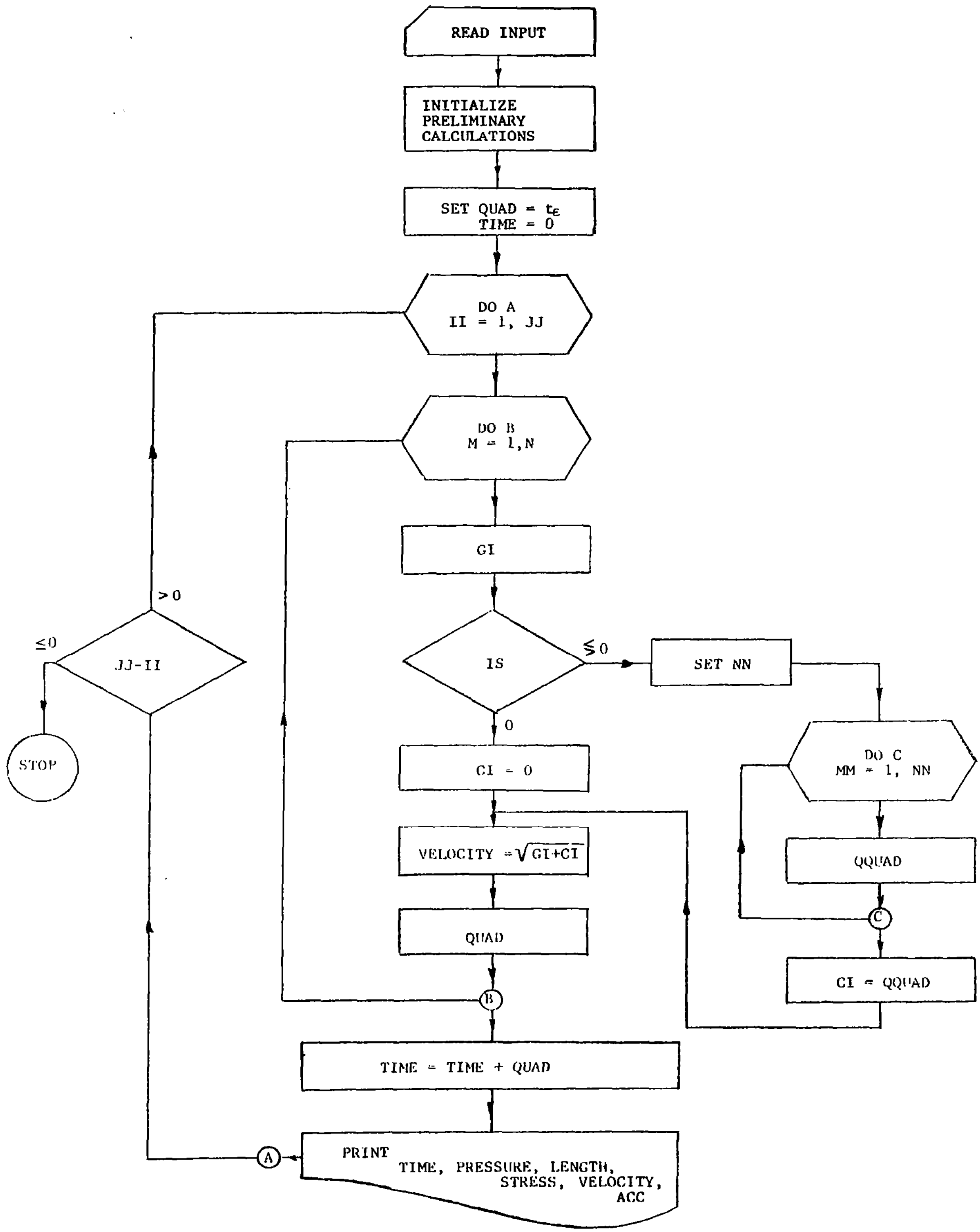
CLO corresponds to the text variable L_o.

Note: The program operates in the C.G.S. system of units, however for convenience, the input and output are in more familiar units.

BLOCK DIAGRAM NOTATION



MAIN DEPLOYMENT PROGRAM



FIRST STAGE COMPUTER PROGRAM LISTING

```
C       TIME FOR FIRST STAGE BAL.   OPENING
      DIMENSION G(20) ,CL(200)
   30 READ (5,1)POR,PCC,E,DC,HS,W,D1,TP,  NAF, NPF,JJ,N,IS,
     1NIM,GAM
    1 FORMAT(8F10.5/3I5/3I5,F10.5)
       JJJ=JJ+1
      READ (5,200)(CL(J),J=2,JJJ)
  200  FORMAT(8F10.5)
      WRITE(6,20)POR,PCC,E,DC,HS,W,D1,TP, NAF,NPF,JJ,N,IS,
     1 NIM,GAM
   20 FORMAT(1H1,4HPOR=,E16.8,2X,4HPCC=,E16.8,2X,2HE=,E16.8,2X,3HDC=,E16
     1.8,2X,3HHS=,E16.8,2X/1H0,2HW=,E16.8,2X,3HD1=,E16.8,2X,3HTP=,E16.8
     2,2X,4HNAF=,I5,2X,4HNPF=,I5,2X,3HJJ=,I5/1H0,2HN=,I5,2X,3HIS=,I5,2X,
     3 4HNIM=,I5,2X,4HGAM=,E16.8)
       PI=3.14159265
      FNAF=NAF
      POR= 1334.*POR
      PCC= 1334.*PCC
      FNPF=NPF
       DPF=24.*PI*D1/FNPF
      CLO= DC*2.54/(2.)
      P4LBM = 4.*PI*CLO/(W*453.6)
      F1=.2984552*PI**3*E*D1*HS/(3.*FNPF*FNPF)*5337.921
      AA=D1*SQRT(2./(FNPF*FNAF)) *30.48
      FB1= F1/(2.*AA*AA*POP)
      PB = PCC/POR
      CLFL=23.940*D1
      DO 995 K1=2,JJJ
  995 CL(K1)= 23.940*D1*CL(K1)
      WRITE(6,996)FB1,PB
  996 FORMAT(1H0,4HFB1=,E16.8,2X,3HPB=,E16.8/////////)
      AZ =+.49614662
      BZ =+2.63150274
      CZ =+7.05571611
      DZ =+5.74280591
      EZ =+1.82244592
      CCC=PI**3*E*D1*FNPF*FNAF**2*HS**3/96.*(.0344374)
      C11=.2984552
      ALP = .45694658
       TIME = 0.0
```

```
      G(1)=5.
      G(2)=1.
      G(3)=6.
      G(4)=1.
      G(5)=5.
      G(6)=2.
      G(7)=1.
      FLN=N
      SM=N*453.6
       EPS= GAM*CLO
       CL(1)= CLO +EPS
        PBAR= POR + PCC
        DO 302 II=1,JJ
      IF(II-1) 301,301,300
301    QUAD= SQRT (EPS*SM/(PI*PBAR*AA*AA*2.))
       GO TO 302
300      QUAD=0.0
       GO TO 302
302      D=CL(II)
        H=(CL(II+1)-CL(II))/(6.*FLN)
 15    DO 100 M=1,N
       DO 100 L =1,7
      FLM= M
       FLL= L
      IF(1-M)21,31,21
 21     T=0.0
       GO TO 99
 31     IF(7-L)21,41,21
 41       T=6.*FLN +1.0
          GO TO 99
 99    TAU =   D+H*(FLM*6.-FLL+T )
       FLG=ALOG(TAU/CLO)
      PRESS= POR*CLO/TAU+ PCC
      FLBL1 = TAU/CLFL
      FLBLO = TAU/CLO
      FLOBL  = CLO/TAU
      FLOL1 = CLO/CLFL
      GIPT=  2.*AA*AA*(POR*FLG+PCC*(FLBLO-1.))* P4LPM
      IF (ALP*CLFL-TAU)250,251,251
251 GIET=P4LPM*CCC*C11*(1.-FLOBL   )**2 /(2.*CLO*CLO)
```

```
     GO TO 252
250 GIET=P4LBM*CCC/(CLO*CLO)*(C11*.5*(1.-FLOL1/ALP)**2-.5*AZ*FLOBL**2
   1*((FLBL1/ALP)**2-1.)  +(AZ+BZ*FLOL1)*(FLOBL)*(FLBL1/ALP-1.)
   2-(BZ+CZ*FLOL1)*FLOL1*ALOG(FLBL1/ALP) +(CZ+DZ*FLOL1)*(FLOL1)*
   3(FLBL1- ALP )      -.5*(DZ+EZ*FLOL1)*FLOL1*
   4 (FLBL1**2-ALP**2) + EZ/3. *FLOL1*(FLBL1**3-ALP**3))
252  GI =GIPT + GIET
     IF(TS)201,202,201
202 CI=0.0
     IF(1-M) 305,306,305
306   IF(7-L) 305,307,305
307 PRTOR=PRESS/1334.
    XLBL1= CL(II+1)/CLFL
    XLBLO= CL(II+1)/CLO
    XLOBL= CLO/CL(II+1)
    IF(ALP*CLFL-CL(II+1))255,256,256
256 ACC=2.*PI/(W*453.6)*(CCC*C11/(CL(II+1)**2)*(1.-XLOBL) +2.*AA*AA*(P
   1OR*XLOBL  +PCC))/30.48
    GO TO 257
255 ACC=2.*PI/(W*453.6)*(CCC*(AZ - BZ*XLBL1+CZ*XLBL1**2- DZ*XLBL1**3
   1+ EZ*XLBL1**4)/(CL(II+1)**2) *(1.-XLOBL)  +2.*AA*AA*(POR*XLOBL +PC
   2C))/30.48
257 CONTINUE
    STRES=PRESS*AA*CL(II+1)*.0057102/(CLFL*HS)
    WRITE(6,308)PRTOR,STRES,ACC
308 FORMAT(1H0,9HPRESSURE=,E16.8,3X,6H(TORR),5X,7HSTRESS=,E16.8,3X,5H(
   1PSI)/1H0,4HACC=,E16.8,3X,12H(FT/SEC.SQ.))
     GO TO 305
201  FNIM= NIM
     NN=FNIM*(TAU/CL(JJ+1))*(2.- TAU/CL(JJ+1)) +2.
    FLNN=NN
    HH=(TAU-CLO)/(6.*FLNN)
    OQUAD =0.
    DO 101 MM=1,NN
    DO 101 LL=1,7
     FLMM=MM
     FLLL=LL
      IF(1-MM)22,32,22
 22    TT=0.0
        GO TO 98
```

```
32       IF(7-LL)22,42,22
42     TT=6.*FLNN+1.0
      GO TO 98
 98 ALP=D+HH*(FLMM*6.-FLLL+TT)
       CII=P4LHM*0.0
     PRESS= POR*CLO/TAU+ PCC
 55 IF(7-L)66,77,66
 77    IF(M-1)66,88,66
 88 IF(7-LL)66,310,66
310 IF(MN-1)66,311,66
311 PRTOR=PRESS/1334.
     XLBL1= CL(II+1)/CLFL
     XLBLO= CL(II+1)/CLO
     XLOBL= CLO/CL(II+1)
     IF(ALP*CLFL-CL(II+1))258,259,259
259 ACC=2.*PI/(W*453.6)*(CCC*C11/(CL(II+1)**2)*(1.-XLOBL) +2.*AA*AA*(P
   1OR*XLOBL +PCC))/30.48
     GO TO 260
258 ACC=2.*PI/(W*453.6)*(CCC*(AZ - BZ*XLBL1+CZ*XLBL1**2- DZ*XLBL1**3
   1+ EZ*XLBL1**4)/(CL(II+1)**2) *(1.-XLOBL)  +2.*AA*AA*(POR*XLOBL +PC
   2C))/30.48
260 CONTINUE
     STRES=PRESS*AA*CL(II+1)*.005702/(CLFL*HS)
     WRITE(6, 39)PRTOR,STRES,ACC
 39 FORMAT(1H0,9HPRESSURE=,E16.8,3X,6H(TORR),5X,7HSTRESS=,E16.8,3X,5H(
   1PSI)/1H0,4HACC=,E16.8,3X,12H(FT/SEC.SQ.))
     GO TO 66
 66 IF(LL-6)151,150,152
151 QQ=1.0
      GO TO 101
150 IF(MM-1)151,153,151
153  QQ=.5
     GO TO 101
152  IF(MM-1)154,151,154
154  QQ=0.0
     GO TO 101
101  QQUAD=QQUAD  +3.*HH/10.*(QQ* G(LL))*CII
     CI=QQUAD
305   F=1./SQRT (GI+CI)
      IF(1-M)6,3,6
```

```
  3  IF(7-L)6,5,6
  5  VEL = 1./F
     VELFS=VEL/30.48
     WRITE(6,38)VELFS
 38  FORMAT(1H ,9HVELOCITY=,E16.8,3X,8H(FT/SEC))
      GO TO 6
  6  IF(L-6)161,160,162
161  Q=1.0
     GO TO 100
160  IF(M-1)161,163,161
163   Q=.5
      GO TO 100
162  IF(M-1)164,161,164
164   Q=0.0
     GO TO 100
100   QUAD=3.*H/10.*(Q*G(L))*F+QUAD
      TIME = QUAD + TIME
     CURLG=CL(II+1)/30.48
     WRITE(6,10)TIME,CURLG
 10  FORMAT(1H-,5HTIME=,E16.8,2X,5H(SEC),4X,7HLENGTH=,E16.8,2X,4H(FT))
102   CONTINUE
      GO TO 30
     END
```

Computer Program for the Inflation Stage

1. Purpose.- The purpose of the program is to provide a solution of the differential equation

$$\bar{m}\ddot{x} = p\ x \sin \phi - F_\theta + \frac{d}{ds}(F_\phi\ x \cos \phi) \tag{39}$$

where

$$F_\theta = F_\theta(x) \tag{45}$$

$$F_\phi = \frac{1}{x \sin \phi}\left[\frac{px^2}{2} - \int_0^s \bar{m}\ \ddot{y}\ ds\right] \tag{44}$$

subjected to the constraints

$$dx^2 + dy^2 = ds^2 \tag{42}$$

$$x \leq x_1 \tag{42}$$

independent of time and the initial and boundary conditions

$$x(t=0,s) = x_o(s) \tag{140}$$

$$y(t=0,s) = y_o(s) \tag{141}$$

$$\dot{x}(t=0,s) = \dot{y}(t=0,s) = 0 \tag{142}$$

$$x(t,s=0) = \dot{x}(t,s=0) = \ddot{x}(t,s=0) = 0 \tag{143}$$

$$y(t,s=\tfrac{\pi}{2}R_1) = \dot{y}(t,s=\tfrac{\pi}{2}R_1) = \ddot{y}(t,s=\tfrac{\pi}{2}R_1) = 0 \tag{144}$$

2. Method.- In order to carry out the numerical solution of the problem we substitute the continuous system by a discrete system of masses m_k interconnected by rigid, massless links, which are determined in the following way.

Taking into account that the area of the spherical surface between two parallels is proportional to the distance between the parallel planes we divide the radius R_1 of the full sphere into an integer number K. The mass of each portion per unit angle of parallel will then be

$$m^* = \frac{1}{2\pi}\ \frac{M}{2K} \tag{145}$$

where M is the total mass of the balloon (including the chemicals). Assuming that the mass between two successive division points, k and k+1, is linearly distributed along the chord, the mass m^* can be substituted by the two masses

$$m_k^* = \frac{m^*}{3} \frac{2x_{1,k} + x_{1,k+1}}{x_{1,k} + x_{1,k+1}} \Delta S_{k,k+1}$$

applied at point k, and

$$m_{k+1}^* = \frac{m^*}{3} \frac{x_{1,k} + 2x_{1,k+1}}{x_{1,k} + x_{1,k+1}} \Delta S_{k,k+1}$$

applied at point k+1. Where

$x_{1,k}$, $x_{1,k+1}$ are the radii of the parallel circles of points k and k+1 in the full sphere.

$\Delta S_{k,k+1}$ is the length of the chord joining points k and k+1 in the full sphere.

The total mass applied at point k will then be:

$$m_k = \frac{m^*}{3} \frac{x_{1,k-1} + 2x_{1,k}}{x_{1,k-1} + x_{1,k}} \Delta S_{k-1,k} + \frac{m^*}{3} \frac{2x_{1,k} + x_{1,k+1}}{x_{1,k} + x_{1,k+1}} \Delta S_{k,k+1}$$

where the first term in the right hand side is the contribution from the mass m^* between points k-1 and k and the second term the contribution from the mass m^* between points k and k+1. Finally

$$m_k = \frac{M}{12\pi K} \left[\frac{x_{1,k-1} + 2x_{1,k}}{x_{1,k-1} + x_{1,k}} \Delta S_{k-1,k} + \frac{2x_{1,k} + x_{1,k+1}}{x_{1,k} + x_{1,k+1}} \Delta S_{k,k+1}\right] \tag{146}$$

On account of symmetry we need to consider only one quadrant. In particular, we have

$$m_o = \frac{M}{12\pi K} \Delta S_{o,1}$$

for the mass at the pole and

$$m_K = \frac{M}{12\pi K} \frac{x_{1,K-1} + 2R_1}{x_{1,K-1} + R_1} \Delta S_{K-1,K}$$

for the mass at the equator with $R_1 = x_{1,K}$. Analogous considerations yield for the horizontal component of the pressure acting at point k the value:

$$H_k = \frac{p}{6} \left[(x_{k-1} + 2x_k)(y_{k-1} - y_k) + (2x_k + x_{k+1})(y_k - y_{k+1})\right] \tag{147}$$

where

x_{k-1}, x_k, x_{k+1} are the radii of the parallel circles of points k-1, k, k+1 in the balloon at time t.

y_{k-1}, y_k, y_{k+1} are the distances to the equatorial plane of the points k-1, k, k+1 in the balloon at time t.

p is the total internal pressure in the balloon at time t.

Under the same assumptions, the distance from the centroid of the element $k, k+1$ to the polar axis will be

$$\bar{x}_{k,k+1} = x_k + \frac{2x_{1,k} + x_{1,k+1}}{3(x_{1,k} + x_{1,k+1})} (x_{k+1} - x_k) \tag{148}$$

in the deformed state.

The total pressure acting on the corresponding parallel plane will be

$$V_{k,k+1} = p \pi \bar{x}^2_{k,k+1}$$

and the membrane force in the meridian direction acting on the element $k, k+1$ will have a vertical component per unit angle

$$F_{\phi_{k,k+1}} \bar{x}_{k,k+1} \frac{y_k - y_{k+1}}{\Delta S_{k,k+1}} = \frac{1}{2} p \bar{x}^2_{k,k+1} - \sum_{r=o}^{k} m_r \ddot{y}_r \tag{149}$$

Hence, its horizontal component will be

$$\overline{F}_{k,k+1} = F_{\phi_{k,k+1}} \overline{x}_{k,k+1} \frac{x_{k+1} - x_k}{\Delta S_{k,k+1}} \left[\frac{1}{2} p \overline{x}^2_{k,k+1} - \sum_{r=o}^{k} m_r \ddot{y}_r \right] \tag{150}$$

Under the same assumptions, the resultant hoop force acting at point k will be:

$$F_{\theta k} = \overline{F}_\theta(x_k) = \frac{y_{k-1} - y_k}{(x_k - x_{k-1})^2} \int_{x_{k-1}}^{x_k} (x - x_{k-1}) F_\theta(x) dx +$$

$$+ \frac{y_k - y_{k+1}}{(x_{k+1} - x_k)^2} \int_{x_k}^{x_{k+1}} (x_{k+1} - x) F_\theta(x)\, dx$$

Assuming that $F_\theta(x)$ varies linearly in each interval we obtain finally

$$F_{\theta k} = \overline{F}_\theta(x_k) = \frac{y_{k-1} - y_k}{6} \left[F_\theta(x_{k-1}) + 2F_\theta(x_k) \right]$$

$$+ \frac{y_k - y_{k+1}}{6} \left[2F_\theta(x_k) + F_\theta(x_{k+1}) \right] \tag{151}$$

where $F_\theta\,(x)$ is given by Equation (79).

The numerical solution is obtained by replacing the differential Equation (140) by

$$\ddot{x}_k = \frac{1}{m_k} \left[H_k - F_{\theta k} + G_k \right] \tag{152}$$

where

$$G_k = \overline{F}_{k,k+1} - \overline{F}_{k-1,k} \tag{153}$$

and the constraint Equation (143) by

$$(x_{k+1} - x_k)^2 + (y_{k+1} - y_k)^2 = \Delta S^2_{k,k+1} \tag{154}$$

To determine the position at time $t_n = t_{n-1} + \Delta t_n$, we assume that the pressure terms remain constant during the time interval Δt_n and calculate:

$$H_k^n = \frac{p_n}{6} \left[(x_{k-1}^{n-1} + 2x_k^{n-1})(y_{k-1}^{n-1} - y_k^{n-1}) + (2x_k^{n-1} + x_{k+1}^{n-1})(y_k^{n-1} - y_{k+1}^{n-1}) \right]$$

$$\overline{P}_{k,k+1}^n = \frac{p_n}{6} \left(\overline{x}_{k,k+1}^n\right)^2$$

with

$$\overline{x}_{k,k+1}^n = x_k^{n-1} + \frac{2x_{1,k} + x_{1,k+1}}{3(x_{1,k} + x_{1,k+1})} \left(x_{k+1}^{n-1} - x_k^{n-1}\right)$$

where

$p_n = p_c + p_o \dfrac{V_o}{V_{n-1}}$	is the internal pressure at time t_{n-1}
p_c	is the constant part of the pressure (sublimation pressure).
p_o	is the initial value of the variable part of the pressure
V_o	is the initial volume
V_{n-1}	is the volume at $t = t_{n-1}$
x_k^{n-1}, y_k^{n-1}	are the coordinates of point k at $t = t_{n-1}$ (closing value of the iteration process for Δt_{n-1}).

It is understood that the term "initial" refers to the beginning of the inflation stage (end of deployment).

As shown in Appendix A, the gases generated by sublimation were not enough to compensate for the change in volume during most of this stage, so a more realistic expression for the pressure will be

$$p_n = p_1 \frac{V_o}{V_{n-1}}$$

where p_1 is the total pressure at the end of the deployment stage.

The iteration process is carried on as:

(a) $x_k^{n,m} = x_k^{n-1} + \dot{x}_k^{n-1}\,\Delta t_n + \ddot{x}_k^{n,m-1}\,\frac{\Delta t_n^2}{2}$

(b) $F_{\theta k}^{n,m} = \bar{F}_{\theta}(x_k^{n,m})$

(c) $y_k^{n,m} = \sum_{r=K}^{k+1} \sqrt{\Delta S_{r-1,r}^2 - (x_r^{n,m} - x_{r-1}^{n,m})^2}$

(d) $\ddot{y}_k^{n,m} = \frac{1}{2}\left[\frac{2}{\Delta t_n^2}\,(y_k^{n,m} - y_k^{n-1} - \dot{y}_k^{n-1}\,\Delta t_n) + \ddot{y}_k^{n,m-1}\right]$

(e) $\bar{\bar{F}}_{k,k+1}^{n,m} = \frac{x_{k+1}^{n,m} - x_k^{n,m}}{y_k^{n,m} - y_{k+1}^{n,m}}\left[\bar{P}_{k,k+1}^{n} - \sum_{r=o}^{k} m_r\,\ddot{y}_r^{n,m}\right]$

(f) $G_k^{n,m} = \bar{\bar{F}}_{k,k+1}^{n,m} - \bar{\bar{F}}_{k-1,k}^{n,m}$

(g) $\ddot{x}_k^{n,m} = \frac{1}{2}\left[\frac{1}{m_k}\,(H_k^n - F_{\theta k}^{n,m} + G_k^{n,m}) + \ddot{x}_k^{n,m-1}\right]$

and repeat steps (a) through (g) until convergence is attained.

In the above expressions

$\dot{x}_k^{n-1} = \dot{x}_k^{n-2} + \ddot{x}_k^{n-1}\,\Delta t_{n-1}$ is the value of the horizontal velocity of point k at $t=t_{n-1}$.

$\dot{y}_k^{n-1} = \dot{y}_k^{n-2} + \ddot{y}_k^{n-1}\,\Delta t_{n-1}$ is the value of the vertical velocity of point k at $t=t_{n-1}$

$\ddot{y}_k^{n-1}$ is the value of the vertical acceleration of point k during the time interval $\Delta t_{n-1}=t_{n-1}-t_{n-2}$ (closing value of the iteration process for Δt_{n-1}).

The first supra index n refers to the time interval under consideration while the second m refers to the iteration cycle.

It was found that the convergence of the iteration process improves markedly by averaging the calculated values with the ones obtained in the previous iteration cycle and hence, steps (d) and (g) were modified accordingly.

The process starts at $t = 0$ by assuming as initial value for the horizontal acceleration

$$\ddot{x}_k^{1,0} = \frac{H_k^1}{m_k}$$

i.e. neglecting the terms $F_{\theta k}$ and G_k as compared to H_k in Equation (127). As shown in Section III, $F_\theta(x)$ attains significance only towards the end of inflation. While the balloon is very elongated in shape (at the beginning of inflation), the meridian forces F_ϕ will be almost parallel to the polar axis and their horizontal component will be small. The pressure p will have its maximum value and its direction will be almost perpendicular to the polar axis, hence, G_k will be small and H_k will be maximum.

It was also found that the convergence of the process was improved by assuming as a starting value the $\ddot{x}_k^{n,o}$ acceleration in the x direction during the n^{th} time interval, the value obtained by linear extrapolation, thus:

$$\ddot{x}_k^{n,0} - \ddot{x}_k^{n-1} = \ddot{x}_k^{n-1} - \ddot{x}_k^{n-2}$$

or

$$\ddot{x}_k^{n,0} = 2\ddot{x}_k^{n-1} - \ddot{x}_k^{n-2}$$

We select the time intervals Δt_n so that the equatorial radius increase approximately by a constant preselected percentage γ during each time interval. To this end we determine the first time interval Δt_1 by setting

$$x_K^{1,0} = x_K^0 + \ddot{x}_K^{1,0} \frac{\Delta t_1^2}{2} = (1 + \gamma)\, x_K^0$$

on account of symmetry and the very elongated initial shape, we have

$$\ddot{x}_K^{1,0} = \frac{H_K^1}{m_K} = \frac{p_o(x_{K-1}^0 + 2x_K^0)}{3m_K} \sim \frac{p_o x_K}{m_K}$$

Hence

$$\Delta t_1 = \sqrt{\frac{2\ \gamma\ x_K^0}{\ddot{x}_K^{1,0}}} = \sqrt{\frac{2\,\gamma\, m_K}{p_o}}$$

where p_o is the value of the internal (residual plus sublimation) pressure at the beginning of the inflation stage.

The subsequent time intervals are determined by the formula

$$\Delta t_n = \gamma\ \frac{t_{n-1}}{\alpha_{n-1}}$$

where

$$\alpha_{n-1} = 2\ \frac{x_K^{n-1} - x_K^{n-2}}{x_K^{n-1} + x_K^{n-2}}$$

is the percentage increase (referred to the mean value) of the equatorial radius during the previous time interval Δt_{n-1}.

Program Symbol	Definition	Text Symbol
DT2	First time interval	Δt_1
DTN1	n-th time interval	Δt_n
X(K), E(K)	Current iterated value of the coordinates	$x_k^{n,m}$, $y_k^{n,m}$
XMP(K)	Previous iterated value of the coordinate	$x_k^{n,m-1}$
XP(K), EP(K)	Value of the coordinates at time $t = t_{n-1}$	x_k^{n-1}, y_k^{n-1}
X1TP(K), E1TP(K)	Velocity components at time $t = t_{n-1}$	$\dot{x}_k^{n-1}$, $\dot{y}_k^{n-1}$
X1TPP(K), E1TPP(K)	Velocity components at time $t = t_{n-2}$	$\dot{x}_k^{n-2}$, $\dot{y}_k^{n-2}$
X2T(K), E2T(K)	Current iterated value of the acceleration components	$x_k^{n,m}$, $y_k^{n,m}$
X2TPM(K), E2TPM(K)	Previous iterated value of the acceleration components	$x_k^{n,m-1}$, $y_k^{n,m-1}$
X2TS(K)	Value of the horizontal acceleration to start the iteration	$x_k^{n,0}$
CPX(K), CPE(K)	Coordinates in the final sphere	$x_{1,k}$, $y_{1,k}$
F(K)	Horizontal component of the meridian force	$\overline{F}_{k,k+1}$
FTH(K)	Hoop Force	$F_{\theta k}$
FMM(K)	Mass	m_k
PN	Internal pressure at time $t = t_n$	p_n

3. Input.- The input consists of:

Balloon parameters:

(a) Total weight of the balloon including the chemicals. (W), lb

(b) Diameter of the fully inflated balloon. (DI), ft

(c) Total thickness of the skin. (HS), mil

(d) Modulus of Elasticity of the skin. (YE), psi

(e) Poisson's ration. (PORT)

(f) Initial weight of sublimation chemicals. (WOO), lb

(g) Inside diameter of the canister. (DIAC), in.

(h) Number of meridian pleat folds. (NPF)

(i) Number of accordion folds. (NAF)

Pressure parameters:

(a) Residual gas pressure at the beginning of the deployment stage (usually set equal to the initial gas pressure in the canister prior to deployment). (PORST), torr

(b) Chemical gas pressure. (PCC), torr
This is the vapor pressure of the subliming compound in solid-vapor equilibrium; a typical value can be obtained from Figure 15 of the reference.[3]

(c) Average temperature of the balloon. (TP), °F

(d) Altitude of the Orbit. (HSS), km

(e) Aspect ratio. (FBB). This is the geometric view factor which represents the relative earth reflected energy incident on the satellite.

(f) Absorptance of the satellite skin to earth radiation. (ALPE)

(g) Absorptance of the satellite skin to Solar radiation. (ALPS)

(h) Chemical latent heat of sublimation. (FLAMS), ergs - $(\text{gm-mole})^{-1}$

(i) Total emittance of the satellite skin. (ESPO)

(j) Chemical molecular weight. (FMOLW), gm/mole

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[3] Clemmons, D. L. Jr., The ECHO I Inflation System, NASA TN D-2194

Control parameters:

(a) Percent increase of the equatorial radius during each time interval (GAM2). This value adjusts the Δt during each time interval such that the equatorial coordinate X(KK) will move outward approximately GAM2 percent. The decimal equilavent of the percent is read in as input (5 percent is read in as 0.05). See 3-C for suggested values and also its relation to KK.

(b) Closing percent change in X coordinates between two successive iteration cycles (CGAM). When the percent change in each X coordinate between two successive iterations reaches this prescribed value, the iteration loop is considered converged; however, the iteration process is forced to go through a minimum number of cycles before it tests the above mentioned coordinate percentage. The decimal equivalent of the percentage is read in as input. A suggested value for CGAM which has resulted in successful runs of the program is 0.001 (i.e. 0.1 percent).

(c) Number of mass point divisions along the meridian is (KK). The magnitude of this number sets geometrical increment of the mesh size for the numerical process, whereas (GAM2) (description 3-a) sets the time increment. Suggested values for these two parameters which have given successful runs for a wide range of problems are: KK=19, GAM2=0.05, however, the program is not strictly required to use these exact values. A range of values which also should give successful runs are:

$$15 \leq KK \leq 35$$

$$0.06 \geq GAM2 \geq 0.025$$

The larger the number of points (KK), the smaller the percentage (GAM2) used about the suggested values KK=19, GAM2=0.05.

(d) Frequency of time printouts (MMM). The total number of time printouts is approximated by:

$$\text{Integer part}\left[\frac{\log_e(NPF\cdot NAF/8)}{GAM2\cdot MMM\cdot 2}\right]+2$$

For example, if every other time calculation (leading up to the final result) is desired, set MMM=5, and for the Pageos sample problem the total number of time printouts would approximately be:

$$\text{Integer part}\left[\frac{\log_e(4441.2)}{0.05\cdot 5\cdot 2}\right]+2 = 18 \text{ Print outs}$$

SUMMARY OF INPUT CARDS FOR INFLATION STAGE

Text Variable	Computer Variable	Definition and Units of Input	Format
W	W	Total weight of Balloon (including the chemicals) (lb)	E 10.5
D_1	D1	Diameter of Inflated Balloon (ft)	E 10.5
h_s	HS	Total Skin Thickness (mils)	E 10.5
E	YE	Modulus of Elasticity of the Skin (psi)	E 10.5
ν	PORT	Poisson's Ratio	E 10.5
W_{co}	WOO	Initial Weight of Sublimation Chemicals (lb)	E 10.5
D_c	DIAC	Inside Diameter of the Canister (in.)	E 10.5
n	NPF	Number of Meridian Pleat Folds	I 5
N	NAF	Number of Accordion Folds	I 5
		END OF FIRST CARD	
P_o	PORST	Residual Gas Pressure (torr)	E 10.5
P_c	PCC	Chemical Gas Pressure (torr)	E 10.5
T	TP	Average Balloon Temperature (°F)	E 10.5
H_s	HSS	Altitude of Orbit (km)	E 10.5
$F_R(\overline{\beta})$	FBB	Aspect Ratio	E 10.5
α_E	ALPE	Absorptance of Satellite Skin to Earth Radiation	E 10.5
α_S	ALPS	Absorptance of Satellite Skin to Solar Radiation	E 10.5
λ_S	FLAMS	Chemical Latent Heat of Sublimation (ergs - $(gm/mole)^{-1}$)	E 10.5
		END OF SECOND CARD	
ε_o	EPSO	Total Emittance of Satellite Skin	E 10.5
M_o	FMOLW	Chemical Molecular Weight (gm/mole)	E 10.5
	GAM2	Percent Equatorial Radius Increase for Each Time Interval	E 10.5
	CGAM	Percent Coordinate Change Between Two Successive Iterations	E 10.5
	KK	Number of Lumped Masses	I 5
	MMM	Frequency of Time Printouts	I 5
		END OF THIRD CARD	

There are, therefore, three cards of required input for the operation of the second stage program.

4. Output.- The first page of output consists of the input which is then readily available for checking and reference purposes.

The second page consists of the starting time, the total pressure (torr) at the start of the second stage inflation, ratio of the starting volume to the final inflated sphere volume, and a list of the initial coordinates which are nondimentionalized in two different ways, that is, division of the coordinate by R_1 (radius of inflated balloon), and secondly, division by the coordinates of the fully inflated sphere. The headings for these variables are respectively, TIME, PRESSURE, VOLUME RATIO, X/R1, E/R1 X/CPX, E/CPE.

The remaining pages of output will be similar, therefore only a typical pair of pages will be described. The number of these intermediate printouts leading up to the final result depends on the input number MMM described in the input section. If the solution converges, the page containing the final result will have the words COMPLETE SOLUTION written at the bottom of the page.

Also listed on the final page only (just above the words COMPLETE SOLUTION) are the maximum membrane stresses which occur at the completion of the inflation process. These three values are respectively, POLAR STRESS, EQUATORIAL MERIDIAN STRESS, EQUATORIAL HOOP STRESS (note only one value is listed for the pole since the polar meridian stress and polar hoop stress are equal). All stresses are in psi units.

The written material on a typical pair of pages consists of the following.

First of the pair:

- The time accumulated during the second stage up to the time of the printout, (it must be emphasized that this particular time value does not include the time accumulated during the first stage)
- The current total pressure (torr)
- The ratio of the current volume to final volume of the fully inflated sphere

- The number of N^{th} time interval and the M^{th} iteration

A list of the current coordinates, nondimensionalized two different ways (i.e. division of the coordinated by R_1 (radius of the inflated balloon, and secondly, division by the coordinates of the fully inflated sphere), and finally a list of the meridian stresses (psi) are tabulated. The headings for these variables respectively are, TIME, PRESSURE, VOLUME RATIO, N, M, X/RI, E/RI, X/CPX, E/CPE, STRESS. The initial coordinates, the results of the first time interval, and the results of the final time interval are printed regardless of the value of MMM.

Computer Variable	Text Variable Notation
X/R1	x/R_1
E/R1	y/R_1
X/CPX	x/x_1
E/CPE	y/y_1

As the sphere inflates in time, X/CPX, E/CPE, and VOLUME RATIO all approach unity.

Second of the pair:

The horizontal and vertical accelerations (ft/sec/sec) and velocities (ft/sec), and vector velocity magnitude are printed. The headings for these variables respectively are, X2T, E2T, X1TP, E1TP, VELM.

Computer Variable	Text Variable Notation
X2T	$\ddot{x}$
E2T	$\ddot{y}$
X1TP	$\dot{x}$
E1TP	$\dot{y}$
VELM	$\sqrt{\dot{x}^2 + \dot{y}^2}$

5. Cutoff.- Upon convergence of the numerical solution, the program arrives at a normal exit and calls for a new problem, at the end of the last problem the program stops. At the end of each converged solution the words COMPLETE SOLUTION are printed.

If the solution does not converge the problem is terminated in one of three ways described here, and then calls for a new problem; at the end of the last problem the program stops.

NOTE-A

This type of failure is indicated on the printout sheet by the words "PROGRAM WILL NOT CONVERGE ON M-CYCLE, SEE PROGRAM WRITEUP, NOTE A, STOPPED AT M = ___, N = ___" indicating the iteration cycle M and the time interval N, at which the failure occurred. This type of failure may be corrected by making CGAM larger. This failure should rarely occur in the usual range of parameters.

NOTE-B

Failure in this case is indicated on the printout sheet by the words "PROGRAM WILL NOT CONVERGE ON N-CYCLE, SEE PROGRAM WRITEUP NOTE B, STOPPED AT M = ___, N = ___. This type of failure should rarely, if ever, occur and would probably be due to erroneous input data.

NOTE-C

Failure here is indicated on the printout sheet by the words "PROGRAM FAILS KA TEST, SEE PROGRAM WRITEUP NOTE C, STOPPED AT M = ___, N = ___." Indication of this type failure means that the ΔX generated was larger than the corresponding arc length ΔS, for a large number of points on the meridian.* This may have happened in one or combinations of the following ways.

(1) GAM2 too large for a given KK or KK to small for a given GAM2, which moves the coordinates out too large a percent, for a given time interval.

(2) GAM2 too small for a given KK or KK to large for a given GAM2, such that numerical roundoff errors in the computer cause failure.

---

* If $\Delta X > \Delta S$ for only a few local points, the TEST BRANCH loop shown on the block diagram makes provisions for this case without failing the entire process.

6. Sample problem.- The PAGEOS balloon will be used as the sample problem for the inflation stage.

Parameter	Computer Variable	Value
Total Weight of Balloon	W	147.5 lb
Diameter of Inflated Balloon	D1	100 ft
Total Skin Thickness	HS	0.5 mils
Modulus of Elasticity of the Skin	YE	0.66×10^6 psi
Poisson's Ratio	PORT	0.475
Initial Weight of Sublimation Chemicals	WOO	10 lb
Inside Diameter of the Canister	DIAC	26.5 in
Number of Meridian Pleat Folds	NPF	85
Number of Accordion Folds	NAF	418
Residual Gas Pressure	PORST	1 torr
Chemical Gas Pressure	PCC	0.003981 torr
Average Balloon Temperature	TP	100°F
Altitude of Orbit	HSS	4250 km
Aspect Ratio	FBB	1.
Absorptance of Satellite Skin to Earth Radiation	ALPE	0.03
Absorptance of Satellite Skin to Solar Radiation	ALPS	0.1
Chemical Latent Heat of Sublimation	FLAMS	0.56×10^{10} ergs-(gm/mole)$^{-1}$
Total Emittance of Satellite Skin	EPSO	0.03
Chemical Molecular Weight	FMOLW	122.12 gm/mole
Percent Coordinate Increase for Each Δt	GAM2	0.05
Percent Coordinate Change Between Two Successive Iterations	CGAM	0.001
Number of Lumped Masses	KK	19
Frequency of Time Printouts	MMM	5

Result: Time for second stage inflation - 5.23 sec

A condensed printout sheet for this sample problem can be found on the following page.

Hence the total time for the first stage deployment plus the time for the second stage inflation is the following sum:

Total Time = 5.15 + 5.23 = 10.38 sec

START OF NEW PROBLEM (INPUT DATA)

BALLOON WEIGHT (INC. CHEM.) (LBS)= 0.14750000E 03

BALLOON DIAMETER (FT)= 0.10000000E 03

SKIN THICKNESS(MILS)= 0.50000000E 00

MODULUS OF ELASTICITY(PSI)= 0.66000000E 06

POISSONS RATIO= 0.47500000E 00

INIT.WT.OF SUBL.CHEMICALS(LBS)= 0.10000000E 02

INSIDE DIAM.OF CANISTER (IN)= 0.26500000E 02

NUMBER OF PLEAT FOLDS= 85

NUMBER OF ACCORD.FOLDS= 418

RESIDUAL GAS PRESSURE (TORR)= 0.10000000E 01

CHEMICAL GAS PRESSURE (TORR)= 0.17760000E-01

BALLOON TEMPERATURE (F)= 0.10000000E 03

ALTITUDE OF ORBIT(KM)= 0.42500000E 04

ASPECT RATIO (F(0)) = 0.10000000E 01

ABSORB.OF SAT.SKIN TO EARTH RADIAT.= 0.30000000E-01

ABSORB.OF SAT.SKIN TO SOLAR RADIAT.= 0.10000000E 00

LATENT HEAT SUBLIMATION(ERGS/GRAM)= 0.56000000E 10

TOTAL EMIT.COEFF.OF SAT.SKIN= 0.30000000E-01

MOLECULAR WEIGHT(GRAMS/MOLE)= 0.12212000E 03

REMAINING ARE PROGRAM CONTROLL VARIABLES

GAMA= 0.50000000E-01

CGAM= 0.10000000E-02

KK= 19

MMM= 5

TIME= 0.00000000E-38 PRESSURE= 0.31853365E-01 VOLUME RATIO= 0.26576739E-03

PT.NUMBER	X/R1	E/R1	X/CPX	E/CPE
1	0.00000000E-38	0.15707963E 01	0.10000000E 01	0.15707963E 01
2	0.49318396E-02	0.12359001E 01	0.14935779E-01	0.13093412E 01
3	0.68743136E-02	0.10949141E 01	0.14954511E-01	0.12328950E 01
4	0.82945435E-02	0.98511076E 00	0.14964338E-01	0.11835645E 01
5	0.94314804E-02	0.89112250E 00	0.14970863E-01	0.11474607E 01
6	0.10378647E-01	0.80700981E 00	0.14975722E-01	0.11194363E 01
7	0.11184360E-01	0.72972764E 00	0.14979598E-01	0.10969573E 01
8	0.11877442E-01	0.65746354E 00	0.14982838E-01	0.10785790E 01
9	0.12476676E-01	0.58903095E 00	0.14985639E-01	0.10634025E 01
10	0.12995052E-01	0.52359877E 00	0.14988124E-01	0.10508384E 01
11	0.13441928E-01	0.46055399E 00	0.14990373E-01	0.10404926E 01
12	0.13824239E-01	0.39942523E 00	0.14992445E-01	0.10321105E 01
13	0.14147221E-01	0.33983690E 00	0.14994381E-01	0.10255563E 01
14	0.14414662E-01	0.28148007E 00	0.14996213E-01	0.10208319E 01
15	0.14630199E-01	0.22409309E 00	0.14997967E-01	0.10181729E 01
16	0.14795517E-01	0.16744808E 00	0.14999663E-01	0.10183095E 01
17	0.14912480E-01	0.11134101E 00	0.15001320E-01	0.10245745E 01
18	0.14982219E-01	0.55584173E-01	0.15002954E-01	0.10576063E 01
19	0.15005393E-01	0.00000000E-38	0.15005393E-01	0.10000000E 01

TIME= 0.52326431E 01 PRESSURE= 0.18332959E-02 VOLUME RATIO= 0.97887176E 00 N= 83 M= 1

CHEMICAL WEIGHT REMAINING= 0.96333543E 01

PT.NUMBER	X/R1	E/R1	X/CPX	E/CPE	STRESS
1	0.00000000E-38	0.10268930E 01	0.10000000E 01	0.10268930E 01	0.21165816E 03
2	0.33020303E 00	0.97080623E 00	0.10000000E 01	0.10284946E 01	0.15541508E 03
3	0.45134911E 00	0.89866421E 00	0.98187332E 00	0.10119137E 01	0.12921068E 03
4	0.53926273E 00	0.83286164E 00	0.97289375E 00	0.10006443E 01	0.10894687E 03
5	0.61254719E 00	0.77400186E 00	0.97231393E 00	0.99664938E 00	0.10283668E 03
6	0.67318463E 00	0.71570142E 00	0.97136224E 00	0.99277870E 00	0.94324045E 02
7	0.72630852E 00	0.65956736E 00	0.97277001E 00	0.99148937E 00	0.91211158E 02
8	0.77212925E 00	0.60368319E 00	0.97400503E 00	0.99035148E 00	0.87186596E 02
9	0.81249502E 00	0.54842034E 00	0.97588150E 00	0.99008641E 00	0.84398063E 02
10	0.84790045E 00	0.49339222E 00	0.97794428E 00	0.99021522E 00	0.81310872E 02
11	0.87915366E 00	0.43663749E 00	0.98042794E 00	0.99097846E 00	0.78331944E 02
12	0.90681252E 00	0.38412275E 00	0.98344199E 00	0.99256900E 00	0.75227468E 02
13	0.93137382E 00	0.32983081E 00	0.98714614E 00	0.99535996E 00	0.71629824E 02
14	0.95315084E 00	0.27568883E 00	0.99159145E 00	0.99932912E 00	0.66992566E 02
15	0.97220516E 00	0.22155711E 00	0.99654405E 00	0.10066507E 01	0.69324411E 02
16	0.98638996E 00	0.16671667E 00	0.10000000E 01	0.10139512E 01	0.89165380E 02
17	0.99407782E 00	0.11113870E 00	0.10000000E 01	0.10227129E 01	0.67753545E 02
18	0.99861795E 00	0.55566980E-01	0.10000000E 01	0.10572792E 01	0.93912416E 02
19	0.10000000E 01	0.00000000E-38	0.10000000E 01	0.10000000E 01	0.93912416E 02

PT.NUMBER	X2T	E2T	X1TP	E1TP	VELM
1	0.00000000E-38	0.00000000E-38	0.00000000E-38	-0.95730795E 01	0.95730795E 01
2	0.00000000E-38	-0.12163579E 03	0.00000000E-38	-0.95730795E 01	0.95730795E 01
3	0.55346642E 01	-0.46184544E 02	0.99596623E 00	-0.16007107E 02	0.16038062E 02
4	-0.60142179E 01	-0.44155526E 02	0.17623537E 01	-0.14949167E 02	0.15011114E 02
5	-0.16586533E 01	-0.42868811E 02	0.24782722E 01	-0.13863454E 02	0.14082345E 02
6	-0.80869257E 01	-0.45940778E 02	0.32725259E 01	-0.12952596E 02	0.13347450E 02
7	-0.81238980E 01	-0.46817896E 02	0.44855514E 01	-0.11900697E 02	0.12717970E 02
8	-0.12873974E 02	-0.48822218E 02	0.57959370E 01	-0.10845802E 02	0.12297329E 02
9	-0.16374964E 02	-0.49854424E 02	0.75182388E 01	-0.96839738E 01	0.12259823E 02
10	-0.19160278E 02	-0.49302835E 02	0.96035734E 01	-0.84526200E 01	0.12793569E 02
11	-0.19191626E 02	-0.46731110E 02	0.12091565E 02	-0.71949263E 01	0.14070285E 02
12	-0.18423718E 02	-0.43169678E 02	0.14969980E 02	-0.59475192E 01	0.16106175E 02
13	-0.20283994E 02	-0.39285343E 02	0.18356372E 02	-0.46908284E 01	0.18946246E 02
14	-0.26599187E 02	-0.34207406E 02	0.22446412E 02	-0.34022593E 01	0.22702792E 02
15	-0.35210451E 02	-0.26266242E 02	0.27311275E 02	-0.21382185E 01	0.27394849E 02
16	-0.41404196E 02	-0.15667847E 02	0.32645496E 02	-0.10594662E 01	0.32662663E 02
17	-0.42215923E 02	-0.58644173E 01	0.37671249E 02	-0.34258837E 00	0.37672407E 02
18	-0.39452181E 02	-0.69129261E 00	0.41372111E 02	-0.36893009E-01	0.41372127E 02
19	-0.37730095E 02	0.00000000E-38	0.42663055E 02	0.00000000E-38	0.42663055E 02

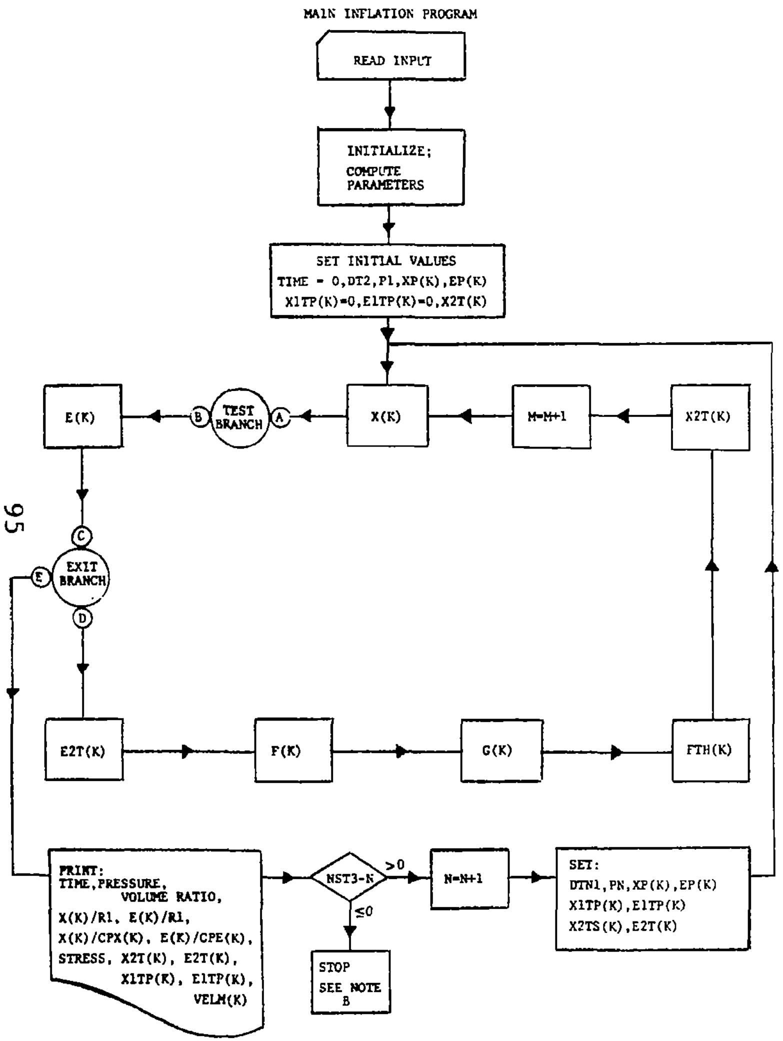
FINAL STRESSES

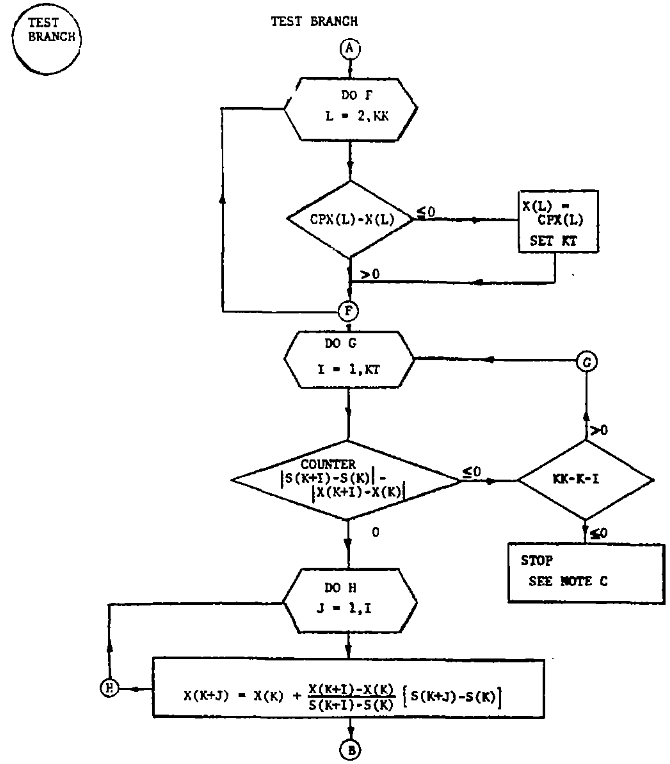
POLAR STRESS (PSI)= 0.23770356E 04

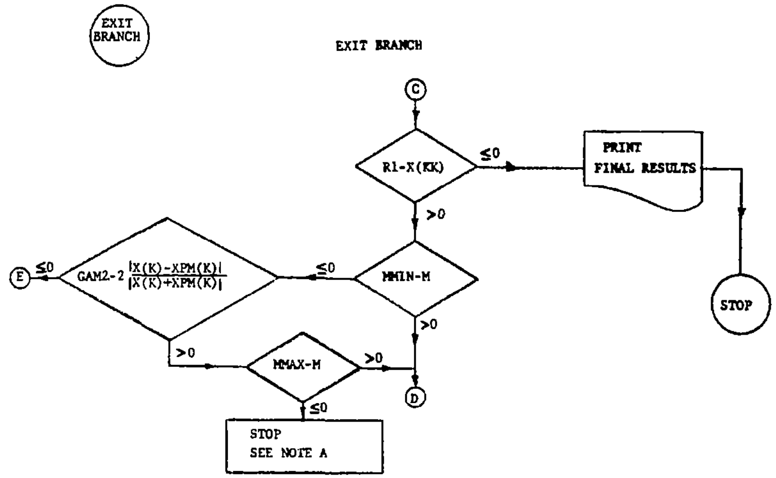
EQ.MER.STRESS (PSI)=, 0.20794641E 04

EQ.HOOP STRESS (PSI)= 0.30232343E 04

COMPLETE SOLUTION


MAIN INFLATION PROGRAM
READ INPUT
INITIALIZE;
COMPUTE PARAMETERS
SET INITIAL VALUES
TIME = 0,DT2,P1,XP(K),EP(K)
X1TP(K)=0,E1TP(K)=0,X2T(K)
E(K)
TEST BRANCH
X(K)
M=M+1
X2T(K)
EXIT BRANCH
E2T(K)
F(K)
G(K)
FTH(K)
PRINT:
TIME,PRESSURE,
VOLUME RATIO,
X(K)/R1, E(K)/R1,
X(K)/CPX(K), E(K)/CPE(K),
STRESS, X2T(K), E2T(K),
X1TP(K), E1TP(K),
VELM(K)
NST3-N
N=N+1
STOP
SEE NOTE B
SET:
DTN1,PN,XP(K),EP(K)
X1TP(K),E1TP(K)
X2TS(K),E2T(K)


TEST BRANCH
TEST BRANCH
DO F
L = 2,KK
CPX(L)-X(L)
X(L) = CPX(L)
SET KT
DO G
I = 1,KT
COUNTER
|S(K+I)-S(K)| - |X(K+I)-X(K)|
KK-K-I
STOP
SEE NOTE C
DO H
J = 1,I
X(K+J) = X(K) + (X(K+I)-X(K))/(S(K+I)-S(K)) [S(K+J)-S(K)]


EXIT BRANCH
EXIT BRANCH
R1-X(KK)
PRINT
FINAL RESULTS
STOP
MMIN-M
GAM2-2 |X(K)-XPM(K)| / |X(K)+XPM(K)|
MMAX-M
STOP
SEE NOTE A

SECOND STAGE COMPUTER PROGRAM LISTING

```
C       SECOND STAGE BALLOON INFLATION   PRESSURE MODIF.
      DIMENSION E(50),X(50),EP(50),XP(50),E2T(50),X2T(50),X1TP(50),Q(50)
     1,F(50),X1TPP(50),FTH(50),G(50),XGG(50),S(50),E1TPP(50),FMM(50),
     2E1TP(50),XMN(50),PHI(50),CPX(50),XMP(50),CPE(50),E2TPM(50),X2TPM(5
     30),X2TPN(50),FMASS(50),X2TS(50),STRES(50),XBP1(50),EPR1(50),XBCPX(
     450),EBCPE(50),NUMB(50),X2TF(50),E2TF(50),X1TPF(50),E1TPF(50),E2T
     5PN(50),VELM(50)
    1 READ(5,2)W,D1,HS,YE,PORT,WOO,DIAC,NPF,NAF,
     1PORST,PCC,TP,HSS,FBB,ALPE,ALPS,FLAMS,
     2ESPO,FMOLW,GAM2,CGAM,KK,MMM
    2 FORMAT(7E10.5,2I5/8E10.5/4E10.5,2I5)
      WRITE(6,20)W,D1,HS,YE,PORT,WOO,DIAC,NPF,NAF
   20 FORMAT(1H1,5X,33HSTART OF NEW PROBLEM (INPUT DATA)///
     11H0,5X,34HBALLOON WEIGHT (INC. CHEM.) (LBS)=,E16.8/
     21H0,5X,22HBALLOON DIAMETER (FT)=,E16.8/
     31H0,5X,21HSKIN THICKNESS(MILS)=,E16.8/
     41H0,5X,27HMODULUS OF ELASTICITY(PSI)=,E16.8/
     51H0,5X,15HPOISSONS RATIO=,E16.8/
     61H0,5X,31HINIT.WT.OF SUBL.CHEMICALS(LBS)=,E16.8/
     71H0,5X,29HINSIDE DIAM.OF CANISTER (IN)=,E16.8/
     81H0,5X,22HNUMBER OF PLEAT FOLDS=,I5/
     91H0,5X,23HNUMBER OF ACCORD.FOLDS=,I5)
      WRITE(6,825)PORST,PCC,TP,HSS,FBB,ALPE,ALPS,FLAMS
  825 FORMAT(
     11H0,5X,29HRESIDUAL GAS PRESSURE (TORR)=,F16.8/
     21H0,5X,29HCHEMICAL GAS PRESSURE (TORR)=,F16.8/
     31H0,5X,24HBALLOON TEMPERATURE (F)=,E16.8/
     41H0,5X,22HALTITUDE OF ORBIT(KM)=,E16.8/
     51H0,5X,21HASPECT RATIO (F(B)) =,F16.8/
     61H0,5X,36HABSORB.OF SAT.SKIN TO EARTH RADIAT.=,E16.8/
     71H0,5X,36HABSORB.OF SAT.SKIN TO SOLAR RADIAT.=,E16.8/
     81H0,5X,36HLATENT  HEAT SUBLIMATION(ERGS/GRAM)=,E16.8)
      WRITE(6,826)ESPO,FMOLW,GAM2,CGAM,KK,MMM
  826 FORMAT(
     11H0,5X,28HTOTAL EMIT.COEF.OF SAT.SKIN=,E16.8/
     21H0,5X,29HMOLECULAR WEIGHT(GRAMS/MOLE)=,F16.8/
     31H0,5X,40HREMAINING ARE PROGRAM CONTROLL VARIABLES./
     41H0,5X,05HGAM2=,E16.8/
     51H0,5X,05HCGAM=,E16.8/
     61H0,5X,03HKK=,I5/
```

```
     71H0,5X,04HMMM=,I5)
      DO 256 JJ=1,KK
      VELM(JJ)=0.0
      E2TPN(JJ)=0.0
      X2TF(JJ)=0.0
      E2TF(JJ)=0.0
      X1TPF(JJ)=0.0
      E1TPF(JJ)=0.0
      E(JJ)=0.0
      X(JJ)=0.0
      EP(JJ)=0.0
      XP(JJ)=0.0
      E2T(JJ)=0.0
      X2T(JJ)=0.0
      X1TP(JJ)=0.0
      S(JJ)=0.0
      F(JJ)=0.0
      X1TPP(JJ)=0.0
      ETH(JJ)=0.0
      G(JJ)=0.0
      XGG(JJ)=0.0
      S(JJ)=0.0
      E1TPP(JJ)=0.0
      EMM(JJ)=0.0
      E1TP(JJ)=0.0
      XMN(JJ)=0.0
      PHI(JJ)=0.0
      CPX(JJ)=0.0
      XMP(JJ)=0.0
      CPE(JJ)=0.0
      E2TPM(JJ)=0.0
      X2TPM(JJ)=0.0
      X2TPN(JJ)=0.0
      FMASS(JJ)=0.0
      X2TS(JJ)=0.0
      STRES(JJ)=0.0
      XBR1(JJ)=0.0
      EBR1(JJ)=0.0
      EBCPE(JJ)=0.0
  256 XBCPX(JJ)=0.0
```

```
      ITRIG=0
      TPK = 255.5 +(5./9.)*TP
      CC1=(1.+.72*(1.-SQRT(1.-(6317./(6317.+HSS))**2)) *(FPB+8.*ALPE/(9.
     1*ALPS)))*(1.3953E+06 *ALPS/FLAMS)
      CC2= ESPO*5.71E-05 *TPK**4/FLAMS
      CC3= 8.3149E+07*TPK/FMOLW
      FM1=W*.1554/(D1*U1)
      R1= 15.24*D1
      PCC=1334.*PCC
      EA= 2.*R1*SQRT(2./(FLOAT(NPF)*FLOAT(NAF)))
      EB=23.939*D1
        PI=3.14159265
      PHI(KK)=PI/2.
       PHI(1)=0.0
       CPX(KK)= R1
       CPX(1)=0.0
      CPE(1)=P1
      CPE(KK)=0.0
      S(1)=0.0
       E2T(KK)=0.0
      E1TP(KK)=0.0
       X2T(1)=0.0
      X1TP(1)=0.0
         KS1=KK-1
       DO 703 J =2,KS1
       FKB=  FLOAT (KK-J)/FLOAT (KK-1)
       EP(J)= R1*ATAN (FKB/SQRT (1.-FKB**2))
  703  XP(J)=  EA*COS (EP(J)/R1)
       XP(KK)=  EA
       EP(1)=  R1*PI*.5
      XP(1)= 0.0
       EP(KK)= 0.0
      NTS3=(ALOG(R1/XP(KK))/GAM2)*1.5
       DO 701 J3=2,KK
  701  S(J3)= S(J3-1) + SQRT ((EP(J3)- EP(J3-1))**2 +(XP(J3)-XP(J3-1))*
     1*2)
      S(KK+1)=2.*S(KK)-S(KK-1)
      DO 700 J2=2, KS1
       AAA=(S(J2)-S(J2-1))*PI/(4.*S(KK))
  700  PHI(J2)=PHI(J2-1) +2.*ATAN (AAA/(SQRT (1.-AAA*AAA)))
```

```
      DO 100 I=1, KK
      G(I)= 0.0
     FTH(I)=0.0
    F1TP(I)=0.
    X1TP(I)=0.
      E(I)=EP(I)
100  X(I)=XP(I)
     DO 709 L=2,KS1
     CPX(L)=CPX(L-1)+(S(L)-S(L-1))*COS ((PHI(L-1)+PHI(L))/2.)
709 CPE(L)=SQRT (R1*R1-CPX(L)*CPX(L))
    DO 257 KF=1,KS1
257 XMN(KF)= 2./3.*(CPX(KF)**2+CPX(KF)*CPX(KF+1)+CPX(KF+1)**2)/(CPX(KF
   1)+CPX(KF+1))
    XMN(KK)=XMN(KK-1)
    DO 950 J6=2,KS1
950   FMM(J6)= FM1/6.*((S(J6)-S(J6-1))*(CPX(J6-1)+2.*CPX(J6))+(S(J6+1)
   1  -S(J6))*(2.*CPX(J6)+CPX(J6+1)))
     FMM(1)=  FM1/6.*S(2)*CPX(2)
     FMM(KK)=  FM1/3.*(S(KK)-S(KK-1))*(CPX(KK-1)+2.*CPX(KK))
    FMR= FM1*P1
    FKK=KK
    VST=0.0
    SPRO=0.0
    DO 960 J7=1,KS1
    SPRO = SPRO+ 2.*(XP(J7)+XP(J7+1))*(EP(J7)-EP(J7+1))
960  VST=VST+(XP(J7)**2+XP(J7+1)**2+XP(J7+1)*XP(J7))*(EP(J7)-EP(J7+1))
     VOR=2.*PI*VST/3.
    VN1=VOR
    SPR=SPRO
    POR= PORST *PI*4.*R1*R1*DIAC*2.54 /(VN1*FLOAT(NPF)*FLOAT(NAF))*
   11334.
    P1 =POR+PCC
     DT2 = SQRT (GAM2*FMR*2./P1)
258   PN=P1
     FMA=0.0
      DO 351 L10=1,KK
      FMA=FMA+FMM(L10)
351    FMASS(L10)=FMA
    TIME= 0.0
    WGT=W00*453.6
```

```
      VSS=0.0
      DO 267 LM=1,KS1
  267 VSS=VSS+(CPX(LM)**2+CPX(LM+1)**2+CPX(LM+1)*CPX(LM))*(CPE(LM)-CPE(
     1LM+1))
      VSP=2.*PI*VSS/3.
      VOLR=VOR/VSP
      DO 970 J8=1,KS1
  970   XGG(J8)=(CPX(J8)+2.*CPX(J8+1))/(3.*(CPX(J8)+CPX(J8+1)))
       XGG(KK)= XGG(KK-1)
      DO 271 LP=1,KK
  271 NUMB(LP)=LP
      DO 268 LP=1,KS1
       EBR1(LP)=E(LP)/R1
  268  EBCPE(LP)=E(LP)/CPE(LP)
      EBCPE(KK)=1.
      DO 269 LQ=2,KK
       XBR1(LQ)=X(LQ)/R1
  269  XBCPX(LQ)=X(LQ)/CPX(LQ)
       XBCPX(1)=1.
      PT=PN/1334.
      WRITE(6,266)TIME,PT,VOLR,(NUMB(JK),XBR1(JK),EBR1(JK),
     1XBCPX(JK),EBCPE(JK),JK=1,KK)
  266 FORMAT(1H1,5HTIME=,E16.8,2X,9HPRESSURE=,F16.8,2X,13HVOLUME RATIO=,
     1E16.8//5X9HPT.NUMBER11X4HX/R113X4HE/R115X5HX/CPX13X5HE/CPE///(9X,I
     22,7X,4(F16.8,2X)))
      DO 101 N=1,NTS3
      VN1P=VN1
      SPRP=SPR
      WGTP=WGT
      PNP=PN
       DO 297 M6=2,KK
      E2TPN(M6)=E2T(M6)
  297    X2TPN(M6)=X2T(M6)
       IF(N-1)5,5,6
    5  DTN1= DT2
       GO TO 7
    6  ALPN1= (X(KK)- XP(KK))/(X(KK)+XP(KK))*2.
       DTN1= GAM2*DTN/ALPN1
    7  DTN =DTN1
      FNN=N
```

```
      JAK=SQRT (FNN+15.01)
      MMAX=3*JAK
      DO 250 KP=1,KK
       X1TPP(KP)=  X1TP(KP)
       E1TPP(KP)  =E1TP(KP)
      EP(KP)=E(KP)
  250  XP(KP)=X(KP)
       DO 112 M=1,MMAX
       DO 888 L1=2,KK
       X2TPM(L1)=X2T(L1)
  888   XMP(L1)= X(L1)
       DO 399 L9=1,KS1
  399   E2TPM(L9)=E2T(L9)
       KZ=1
       KT=2
    8  DO 820 K1=2,KK
  995  IF(XP(K1)-CPX(K1)) 102,821,821
  102   IF(N-1)961,961,962
  962    IF(M-1) 963,963,961
  963  X(K1)=XP(K1)+DTN1*X1TP(K1)+X2TS(K1)*DTN1*DTN1/2.
       GO TO 391
  961    XP(KK+1)=XP(KK-1)
       EP(KK+1)=  -EP(KK-1)
      X2T(K1)=((PN/6.*((EP(K1-1)-EP(K1))*(XP(K1-1)+2.*XP(K1))+(EP(K1)-EP
     1(K1+1))*(2.*XP(K1)+XP(K1+1))))-FTH(K1)+G(K1))/FMM(K1)+X2TPM(K1))/2.
         X(K1)=  XP(K1)+ DTN1*X1TP(K1) + X2T(K1)* DTN1*DTN1/2.
  391  IF(X(K1)-CPX(K1))820,821,821
  821   X2T(K1)=0.0
      IF(ITRIG.EQ.1) GO TO 1001
      ITRIG=ITRIG+1
 1001 X(K1)=CPX(K1)
      IF(K1-KZ-1)731,730,731
  730  KZ=K1
       KT=K1
      GO TO 731
  731  CONTINUE
       GO TO 820
  820   FTH(K1)= 0.0
        DO 800 K12=KT,KK
       KA=KK-K12+KT
```

```
801     DO 830 KB=1,KK
        KAKB=KA+KB-1
       IF((X(KA-1)-X(KAKB))**2-   (S(KA-1)-S(KAKB))**2)828,830,830
830    CONTINUE
828    IF(KB-1) 800,800,829
829    IF(KK+1-KA-KB)991,831,831
831    KBB=KB-1
        DO 840 KC=1,KBB
        KABC= KA+KC-1
        KAB=KA+KBB
840 X(KABC)= X(KAB)-(S(KAB)-S(KABC))*(X(KAB)-X(KA-1))/(S(KAB)-S(
   1KA-1))
        GO TO 800
800 CONTINUE
        DO 104 K3=1,KS1
        KKK=KK-K3
      DSMDX = (S(KKK+1)-S(KKK))**2-(X(KKK+1)-X(KKK))**2
104   E(KKK)= E(KKK+1) +SQRT (DSMDX)
      PSIT=R1-X(KK)
      IF(PSIT)381,381,737
737   CONTINUE
        DO 105 K4=1,KS1
105    E2T(K4)=(( E(K4) - EP(K4)- E1TP(K4)*DTN1)*2./(DTN1*DTN1) +
    1    E2TPM(K4))/2.
798      IF(M-JAK)10,10,11
11    DO 889 L2=2,KK
      TCGAM= ABS ((X(L2)-XMP(L2))*2./(X(L2)+XMP(L2)))-CGAM
       IF (TCGAM)9,9,889
889    CONTINUE
10     Q(1)=  -FMM(1)*E2T(1)
        DO 106 K5=2 ,KK
106    Q(K5)= Q(K5-1) -FMM(K5)*E2T(K5)
      E(KK+1)=-E(KK-1)
        DO 107 K6=2,KS1
107    F(K6)=(X(K6+1)-X(K6))/(E(K6)-E(K6+1))*(PN/2.*(XP(K6)+XGG(K6)*(XP
     1(K6+1)-XP(K6)))**2+Q(K6))
       F(KK)= -F(KK-1)
      F(1)=  X(2)/(E(1)-E(2))*(2.*PN/9.*XP(2)**2-FMM(1)*E2T(1))
      X(KK+1)= X(KK-1)
        DO 108 K7=2,KK
```

```
108    G(K7)=  F(K7)- F(K7-1)
    IF(MMAX-M)200,200,112
112    CONTINUE
  9   DO 109 K8=KZ, KK
109 X1TP(K8)=X1TPP(K8)+X2T(K8)*DTN1
      DO 110 K9=1,KS1
110 E1TP(K9)=E1TPP(K9)+E2T(K9)*DTN1
      IF(K7-1)381,381,382
382  E1A=0.0
     DO 357 L12=1,KZ
     E1A=E1A+ E1TP(L12)*FMM(L12)/FMASS(KZ)
357    E1TP(K7)=E1A
355 DO 261 L7=1,KZ
261   X1TP(L7)= 0.0
     DO 262 L8=1,KZ
262   E1TP(L8)= E1TP(KZ)
381       VST =0.0
    SPR=0.0
       DO 111 K10=1 ,KS1
    SPR= SPR + 2.*(X(K10)+X(K10+1))*(E(K10)-E(K10+1))
111  VST= VST + (X(K10)*X(K10)+X(K10+1)*X(K10+1)+X(K10+1)*X(K10))*(
   1 E(K10)-E(K10+1))
     VN1= 2.*PI*VST/3.
     XDR1= X(KK)/R1
    SSS=PI*(30.48*D1)**2
    DELWT =(CC1*SPR -CC2*SSS)*DTN1
    IF(DELWT.LE.0.0)DELWT=0.0
    IF(WGTP -DELWT) 870,870,871
870 WGT =0.0
    MARK= 3
    BRACE =0.0
    GO TO 875
871  WGT = WGTP - DELWT
    GO TO 872
872  BRC2 = CC3 * DELWT
     BRC1 = PCC*(VN1-VN1P)
     IF(BRC2-BRC1)873,873,874
873  BRACE = BRC2
     MARK= 2
     GO TO 875
```

```
874   BRACE= BRC1
      MARK=1
     GO TO 875
875 PN =( PNP*VN1P +BRACE)/ VN1
      VOLR=VN1/VSP
      TIME=  TIME+ DTN1
     DO 273 LD=1,KS1
273  STRES(LD)=ABS (.00571*F(LD)*(S(LD+1)-S(LD))/((X(LD+1)-X(LD))*XM*
    1(LD)*HS))
     STRES(KK)=STRES(KK-1)
     PSIT=R1-X(KK)
      IF (PSIT)13,13,12
 12    IF(NTS3-N)203,203,14
101    CONTINUE
 14  IF(N-1) 272,13,272
272  IF(ITRIG.EQ.1) GO TO 1005
     IF(MOD(N,MMM))379,13,379
1005  ITRIG=ITRIG+1
  13 DO 275 LF=1,KS1
     EBR1(LF)=E(LF)/R1
 275 EBCPE(LF)=E(LF)/CPE(LF)
     EBCPE(KK)=1.
     DO 276 LG=2,KK
     XBR1(LG)=X(LG)/R1
 276 XBCPX(LG)=X(LG)/CPX(LG)
     XBCPX(1)=1.
     PT=PN/1334.
     IF(PSIT)932,932,933
 932 DO 934 LM=1,KK
     X2T(LM)=X2TPN(LM)
 934 E2T(LM)=E2TPN(LM)
     GO TO 933
 933 DO 930 LS=1,KK
     X2TF(LS)=X2T(LS)/30.48
     E2TF(LS)=E2T(LS)/30.48
     X1TPF(LS)=X1TP(LS)/30.48
     E1TPF(LS)=E1TP(LS)/30.48
 930 VELM(LS)=SQRT(X1TPF(LS)*X1TPF(LS)+E1TPF(LS)*E1TPF(LS))
     WZT  = WGT/453.6
     WRITE(6,307)TIME,PT,VOLR,N,M,WZT,      (NUMB(JJ),XBR1(JJ),EBR1(JJ),
```

```
   1XBCPX(JJ),EBCPE(JJ),STRES(JJ), JJ=1,KK)
307 FORMAT(1H1,5HTIME=,E16.8,2X,9HPRESSURE=,E16.8,2X,13HVOLUME RATIO=,
   1E16.8,2X,2HN=,I4,2X,2HM=,I4
   2//5X,26HCHEMICAL WEIGHT REMAINING=,E16.8
   3     //5X9HPT.NUMBER11X4HX/RI13X4HE/RI15X5HX/CPX13X5HE/CPE12X,6HST
   4RESS///(9X,I2,7X,5(E16.8,2X)))
    WRITE(6,931) (NUMB(JJ),X2TF(JJ),E2TF(JJ),X1TPF(JJ),E1TPF(JJ),VELM
   1(JJ),JJ=1,KK)
931 FORMAT(1H1,5X9HPT.NUMBER11X3HX2T14X3HE2T15X4HX1TP13X4HE1TP13X4HVEL
   1M///(7X,I2,7X,5(E16.8,2X)))
379 IF(PSIT)740,740,295
740 TKE=.5*FMM(KK)*X1TPF(KK)*X1TPF(KK)
    TXM=  FMM(KK)*X1TPF(KK)*.5
    DO 321 I2=1,KS1
    TKE = TKE + FMM(I2)*VELM(I2)**2
321 TXM =TXM +     FMM(I2)*X1TPF(I2)
    TKE = TKE*3.14159*4./(W*453.6)
    TXM = TXM*16./(W*453.6)
    SWOSQ =      8.*(1.+PORT)*3.14159*YF*HS/((1.-PORT**2)*(W/386.4)*100
   10.)
    SWO  =SQRT(SWOSQ)
    OM2SQ= 2.+ 1.5*(1.+PORT) -SQRT(4.+2.*(1.+2.*PORT)*(1.+PORT)+(1.5*(
   1 1.+PORT))**2)
    SW2SQ = OM2SQ*4.*3.14159*YF*HS/((1.-PORT**2)*(W/386.4)*1000.)
    SW2  =SQRT(SW2SQ)
    POT1=1.+PORT
    ALPQ = OM2SQ/POT1
    AAQ = .2*(1.+ (2.- ALPQ)**2) + (.125*(3.-ALPQ))**2
    BBQ = .25*(3.-ALPQ)*TXM
    CCQ = -(TXM**2 -TKE)
    YMS =(-BBQ -SQRT(BBQ**2 + 4.*AAQ*CCQ))/(2.*AAQ)
    XMS = TXM + .125*(3.-ALPQ)* YMS
    CWO =  XMS/SWO
    CW2 =  YMS/SW2
    SPSC = PT*D1*1000.*12.*.01933/(4.*HS)
    SCOF1= YF*2./((1.-PORT**2)*D1)
    SCOF2= ABS(CWO)* POT1
    SCOF3= (2.*POT1-OM2SQ)/POT1
    SIGO = SPSC +SCOF1*(SCOF2 + ABS(CW2)*OM2SQ/2.)
    SIGP = SPSC -SCOF1*(SCOF2 +.5*ABS(CW2*(-POT1+SCOF3)))
```

```
      SIGT = SPSC +SCOF1*(SCOF2 +.5*ABS(CW2*(-POT1+PORT*SCOF3)))
      WRITE(6,741)SIGO,SIGP,SIGT
  741 FORMAT(/////50X,14HFINAL STRESSES////10X,19HPOLAR STRESS (PSI)=,E1
     16.8,///10X,21HEQ.MER.STRESS (PSI)=,E16.8///10X,21HEQ.HOOP STRESS (
     2PSI)=,E16.8///////10X,17HCOMPLETE SOLUTION )
      GO TO 1
  295   IF(K-2)378,294,294
  294   KZZ=KZ+1
        DO 296 M2=KZZ,KK
  296   X2TS(M2)=X2T(M2)*2.-X2TPN(M2)
        IF(KZ-1)101,101,378
  378  E2A=0.0
       DO 352 L11=1,KZ
       E2A=E2A+FMM(L11)*E2T(L11)/FMASS(KZ)
  352   E2T(KZ)=E2A
  354 DO 699 L6=1,KZ
  699  E2T(L6)= E2T(KZ)
       GO TO 101
  200 WRITE(6,201) M,N
  201 FORMAT( 81H1PROGRAM WILL NOT CONVERGE ON M-CYCLE,SEE PROGRAM WRITE
     1 UP ,NOTE A, STOPPED AT M=,I3,2X,2HN=,I3)
       GO TO 1
  203 WRITE(6,204) M,N
  204 FORMAT( 81H1PROGRAM WILL NOT CONVERGE ON N-CYCLE,SEE PROGRAM WRITE
     1 UP ,NOTE B, STOPPED AT M=,I3,2X,2HN=,I3)
       GO TO 1
  991 WRITE(6,997)M,N
  997 FORMAT(66H1PROGRAM FAILS KA TEST,SEE PROGRAM WRITE UP, NOTE C, STO
     1PPED AT M=I3,2X,2HN=,I3)
      GO TO 1
      END
```

APPENDIX C
PARAMETRIC CURVES

The curves in this appendix have been determined for two typical balloons of the following characteristics:

	PAGEOS	ECHO II
Diameter, ft	100.0	135.0
Skin Thickness, mils	0.50	0.71
Total Weight (incl chemicals), lb	147.5	493.8
Canister Diameter, in.	26.5	28.0
No. of Accordion Folds	85.0	85.0
No. of Pleat Folds	418.0	360.0
Modulus of Elasticity, lb $in.^{-2}$	$6.6x10^5$	$2.73x10^6$
Subliming Chemical	Benzoic Acid	Benzoic Acid
Weight of Subliming Chemical, lb	10.0	52.4

Note on the velocity and stress diagrams.- Figures 8 through 16 are plots of velocity versus time and stress versus time for the PAGEOS and ECHO II satellites. It should be noticed, however, that while Figure 8 gives the velocity of the tip (pole) of the balloon during the deployment stage, which happens to be the largest velocity at every instant, Figures 12 and 14 give the greatest of the moduli of the velocities of the points in the meridian, regardless of location.

Figure 9 shows the value of the hoop stress at the equator during deployment, while Figures 13 and 15 give the maximum value of the meridian stress, regardless of position, for the inflation stage. Moreover, while it is apparent that inflation will likely start before the balloon has reached full deployment, the present state-of-the-art does not provide a means of determining exactly when this will occur. Hence, it was considered that the inflation stage started from rest, and the initial shape is the one attained at 100 percent deployment.

Figures 10 and 11 are plots of the tip velocity and meridian stress at the equator that will occur, if the balloon was allowed to reach full deployment and if all the kinetic energy was to be absorbed by the elastic deformation of the skin. This is a conservative estimate that will certainly give stresses larger than the actual ones.

Figure 16 is a representative plot of the stresses after inflation has been completed. The assumption is that all the kinetic energy of the skin is absorbed by the elastic deformation and again gives a conservative estimate as it assumes only two of the numerous modes of vibration, and neglects completely any structural dampling. The values given by the computer program are the absolute maximum that the stresses can attain under these assumptions, regardless of the time. It must be noticed that, as the ratio between the two natural frequencies ω_o and ω_2 is not, in general, a rational number, the time of the absolute maximum will be infinite. On the other hand, the presence of structural damping will have the effect that for a large time the system will come to rest, hence, the calculated values will be conservative.

Table II gives the tip velocity at the end of the deployment stage as obtained from the computer program and also the values of the reference stress

$$\sqrt{\frac{ME}{\pi h_s}} \ \frac{\dot{L}_i}{D_1} \quad \text{(psi)}$$

used to adimensionalize the stresses in Figure 11. Table III gives the absolute maximum value that the stresses can attain after the end of inflation, as obtained from the computer program. These values were used in the plots of Figure 16.

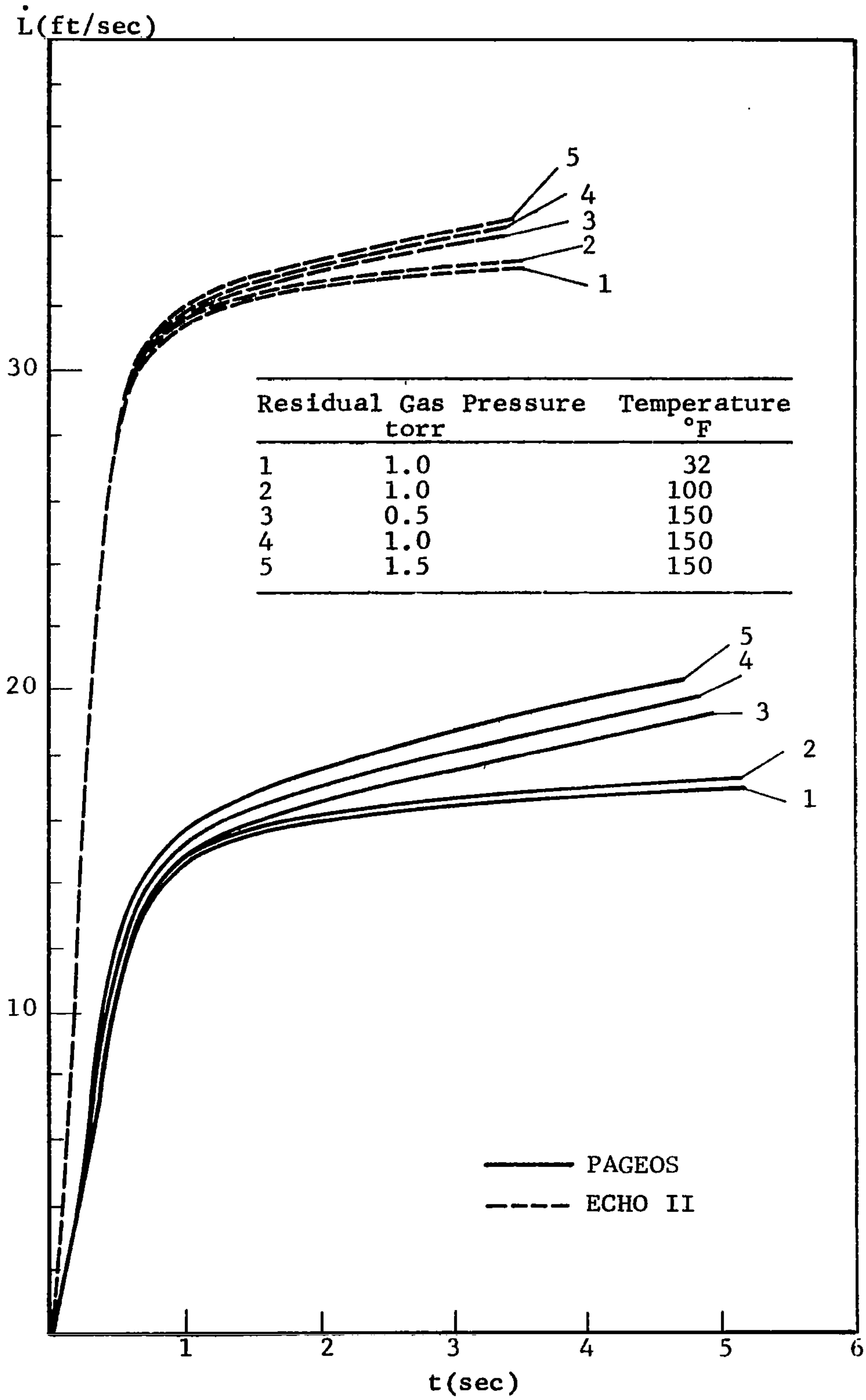


Figure 8 Tip Velocity versus Time for the Deployment Stage

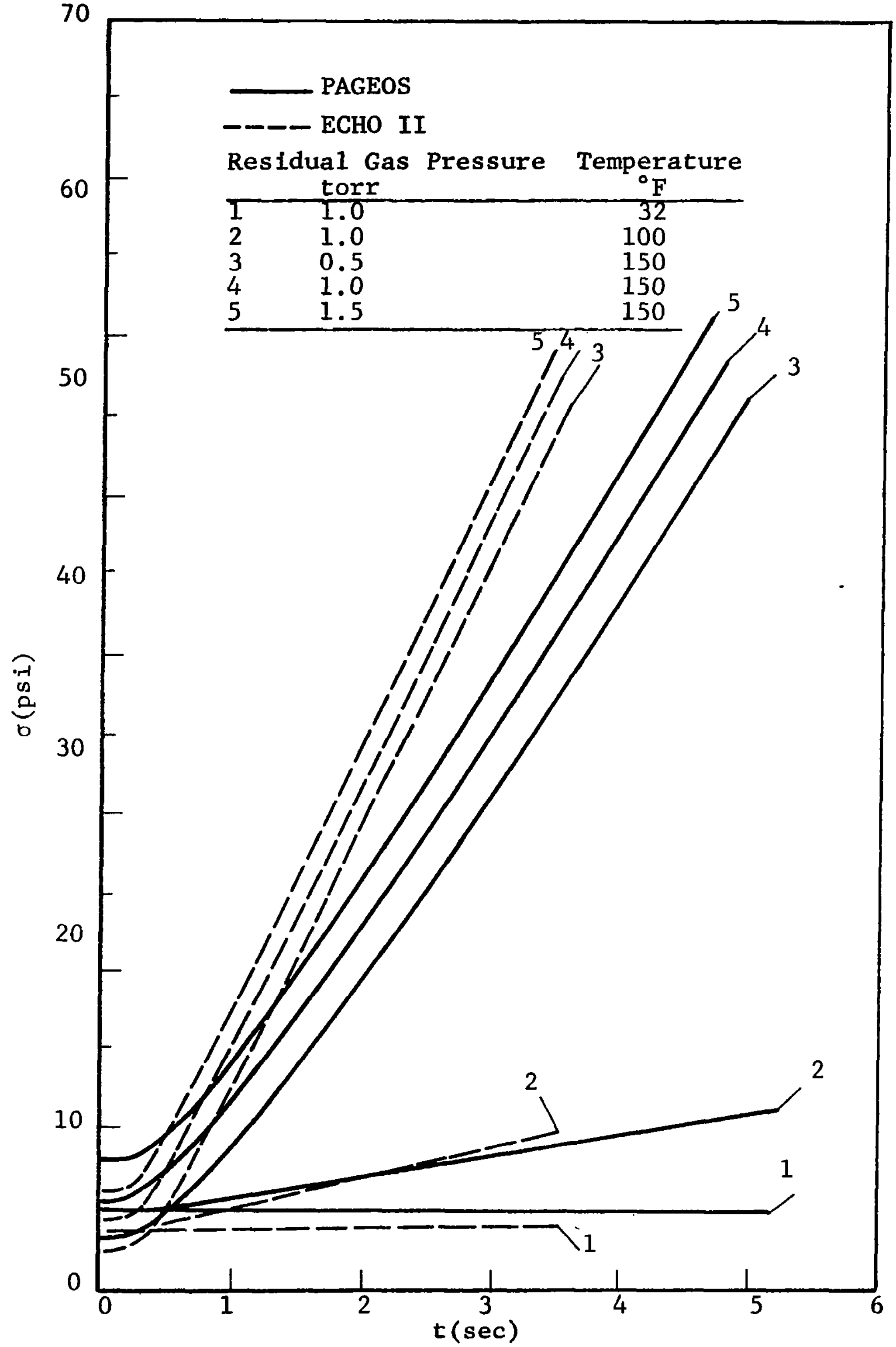


Figure 9 Equatorial Hoop Stress versus Time for the Deployment Stage

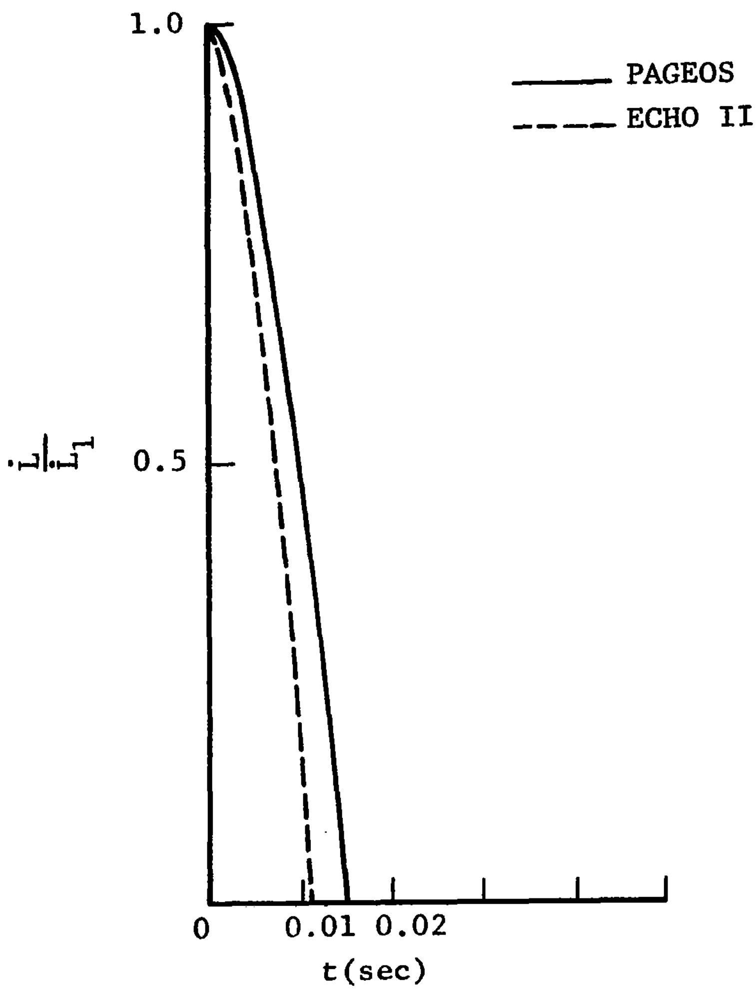


Figure 10 Dimensionless Tip Velocity versus Time at the End of Deployment Stage

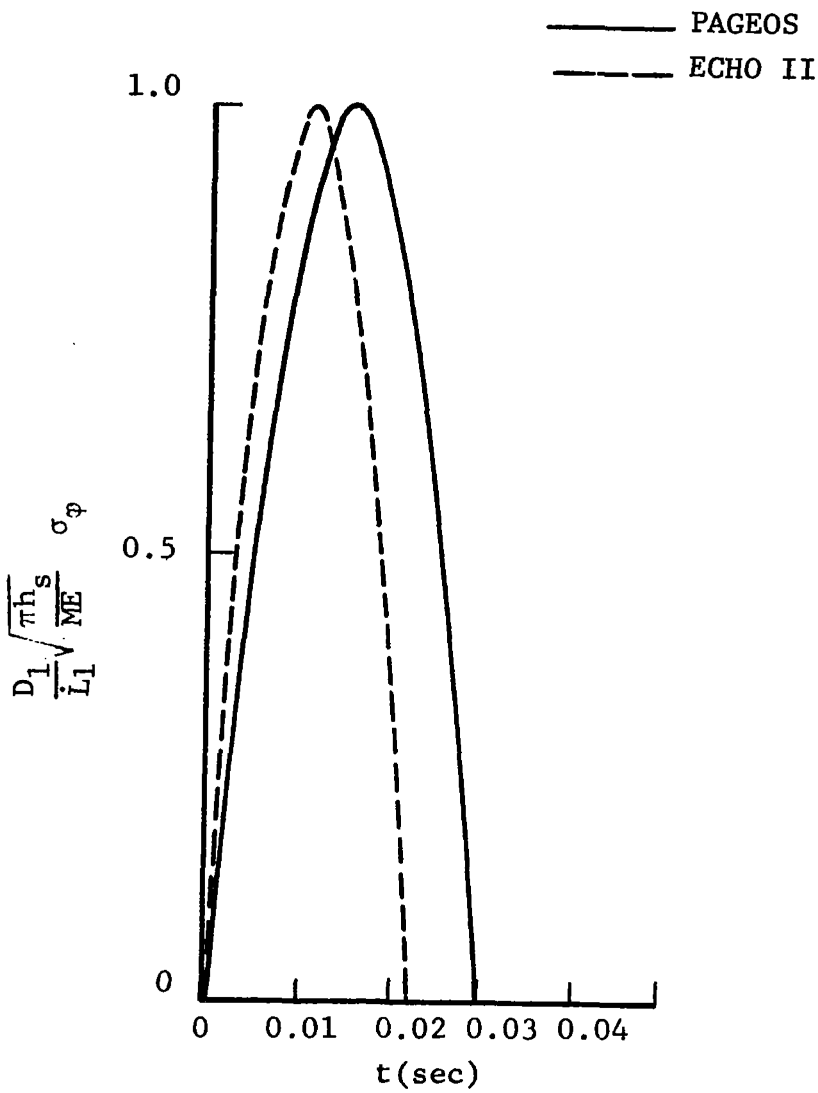
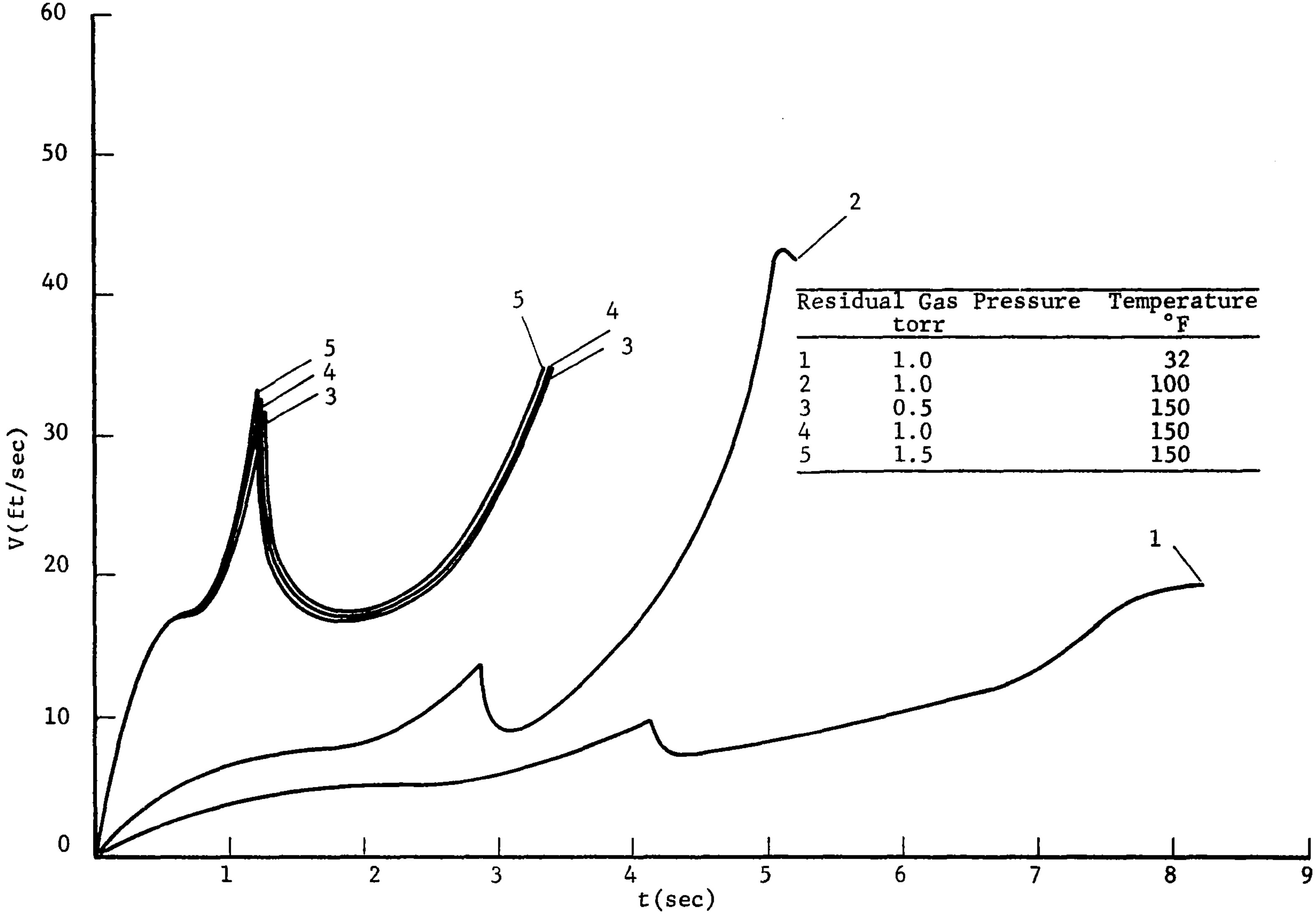


Figure 11 Dimensionless Equatorial Meridian Stress versus Time at the End of Deployment Stage



Residual Gas Pressure torr		Temperature °F
1	1.0	32
2	1.0	100
3	0.5	150
4	1.0	150
5	1.5	150

Figure 12 PAGEOS - Maximum Velocity versus Time for the Inflation Stage

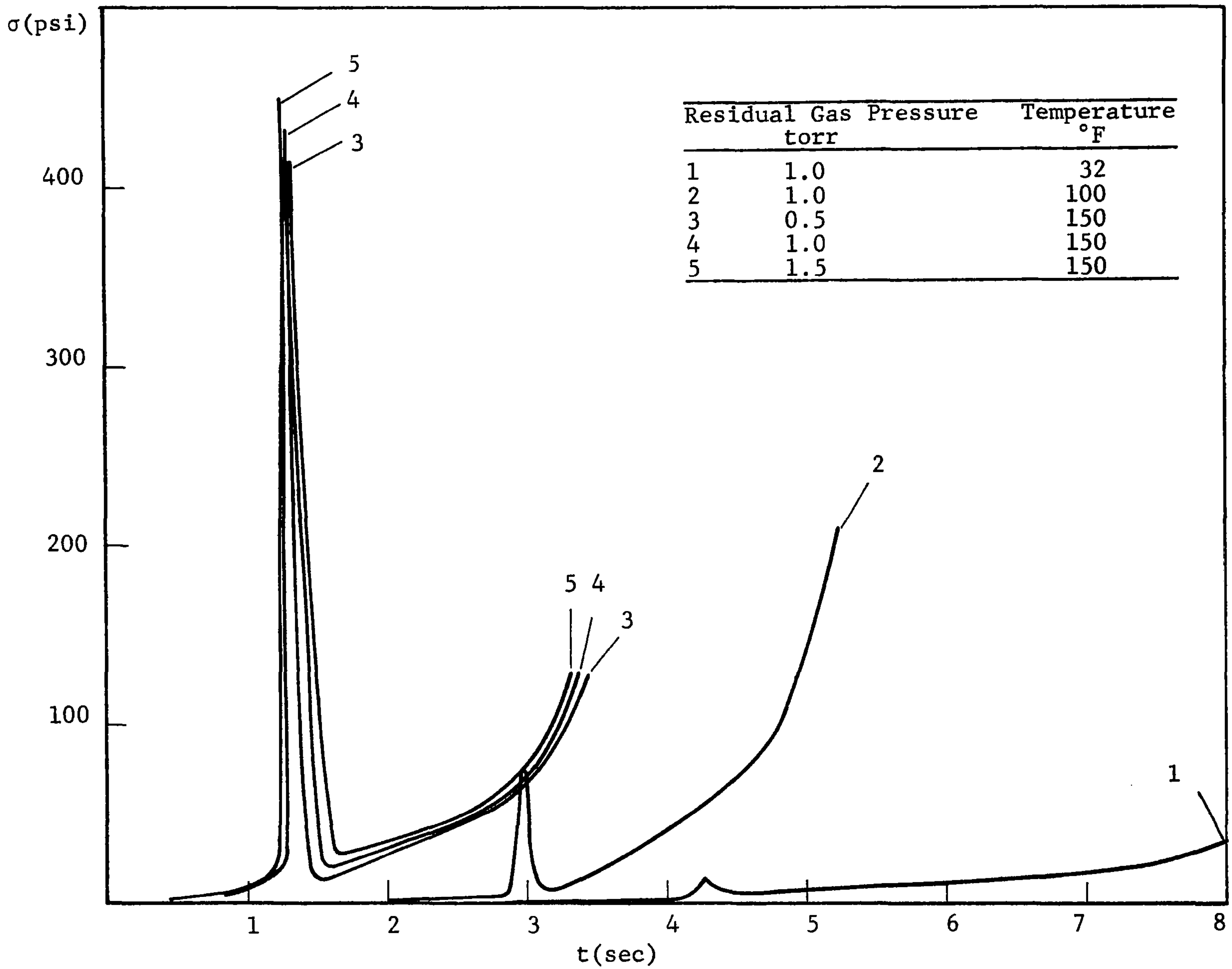
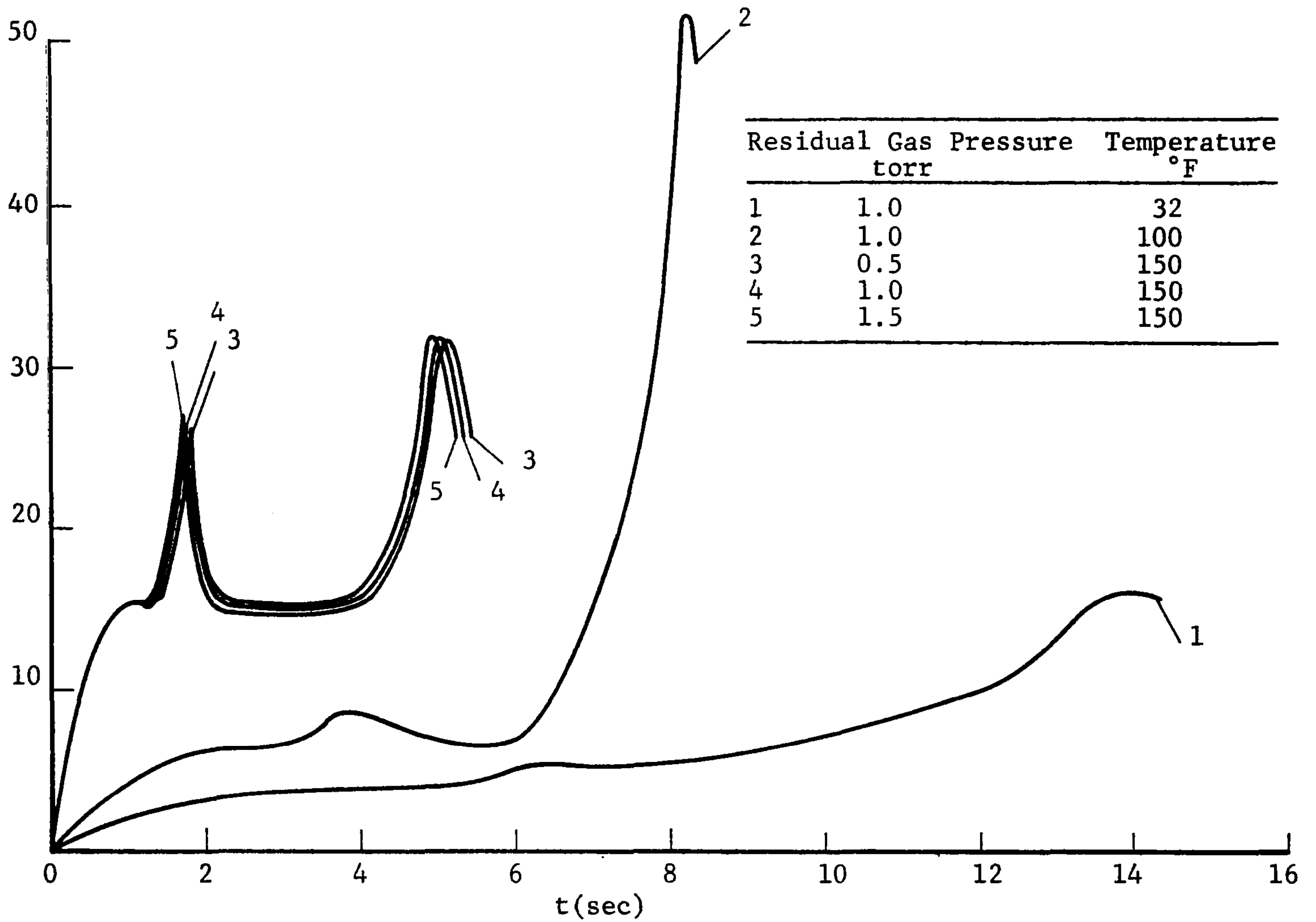


Figure 13 PAGEOS - Maximum Meridian Stress versus Time for the Inflation Stage



	Residual Gas Pressure torr	Temperature °F
1	1.0	32
2	1.0	100
3	0.5	150
4	1.0	150
5	1.5	150

Figure 14 ECHO II - Maximum Velocity versus Time for the Inflation Stage

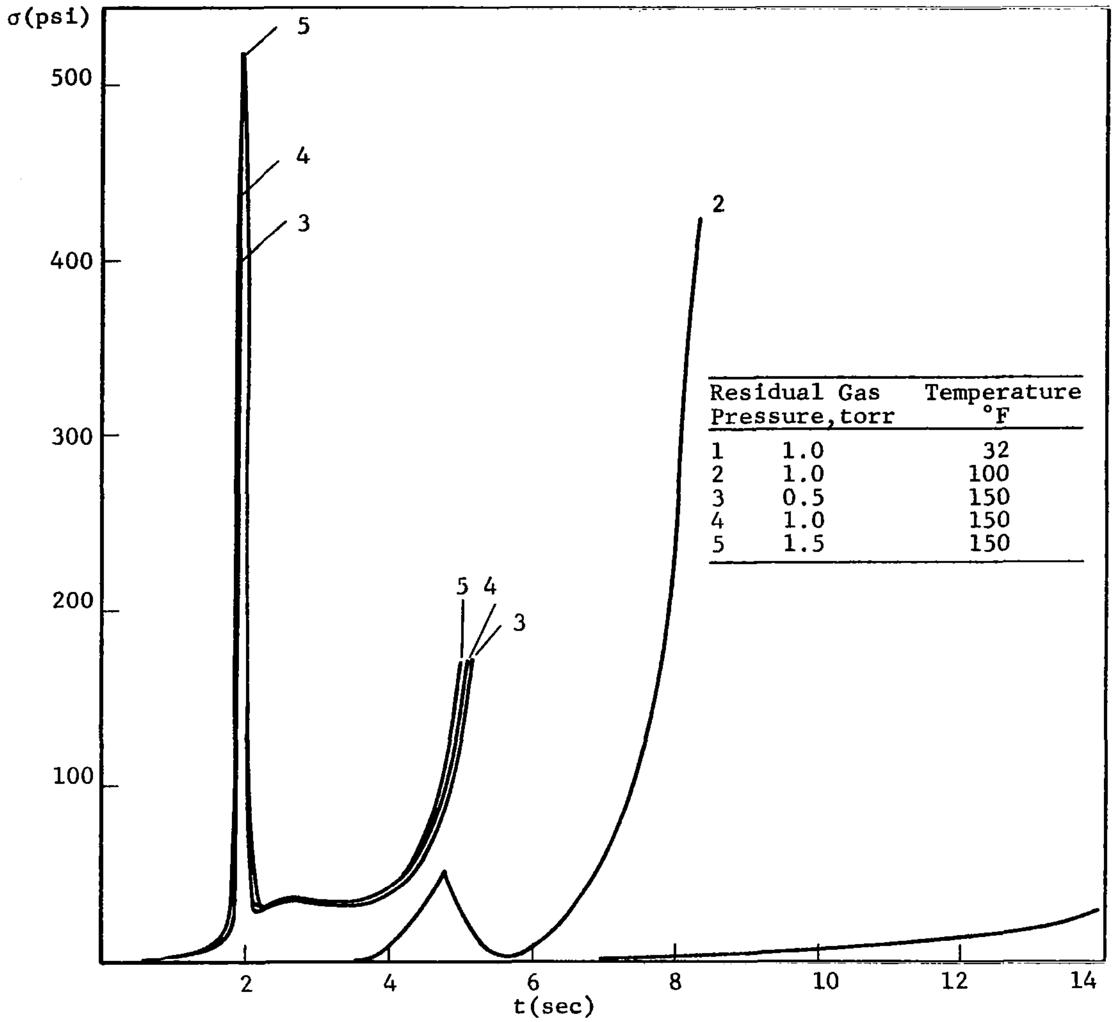


Figure 15 ECHO II - Maximum Meridian Stress versus Time for the Inflation Stage

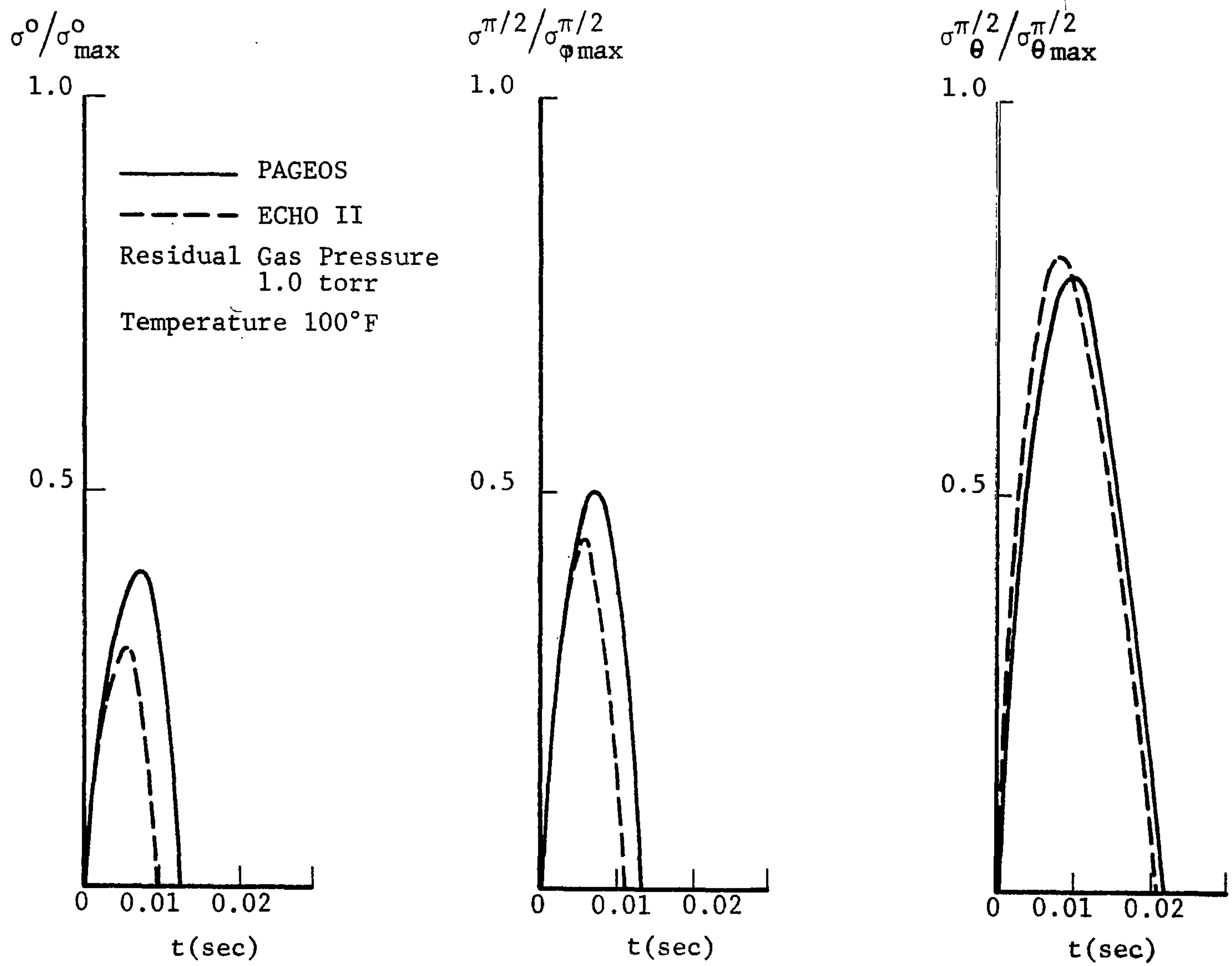


Figure 16 Dimensionless Stresses versus Time at the End of Inflation

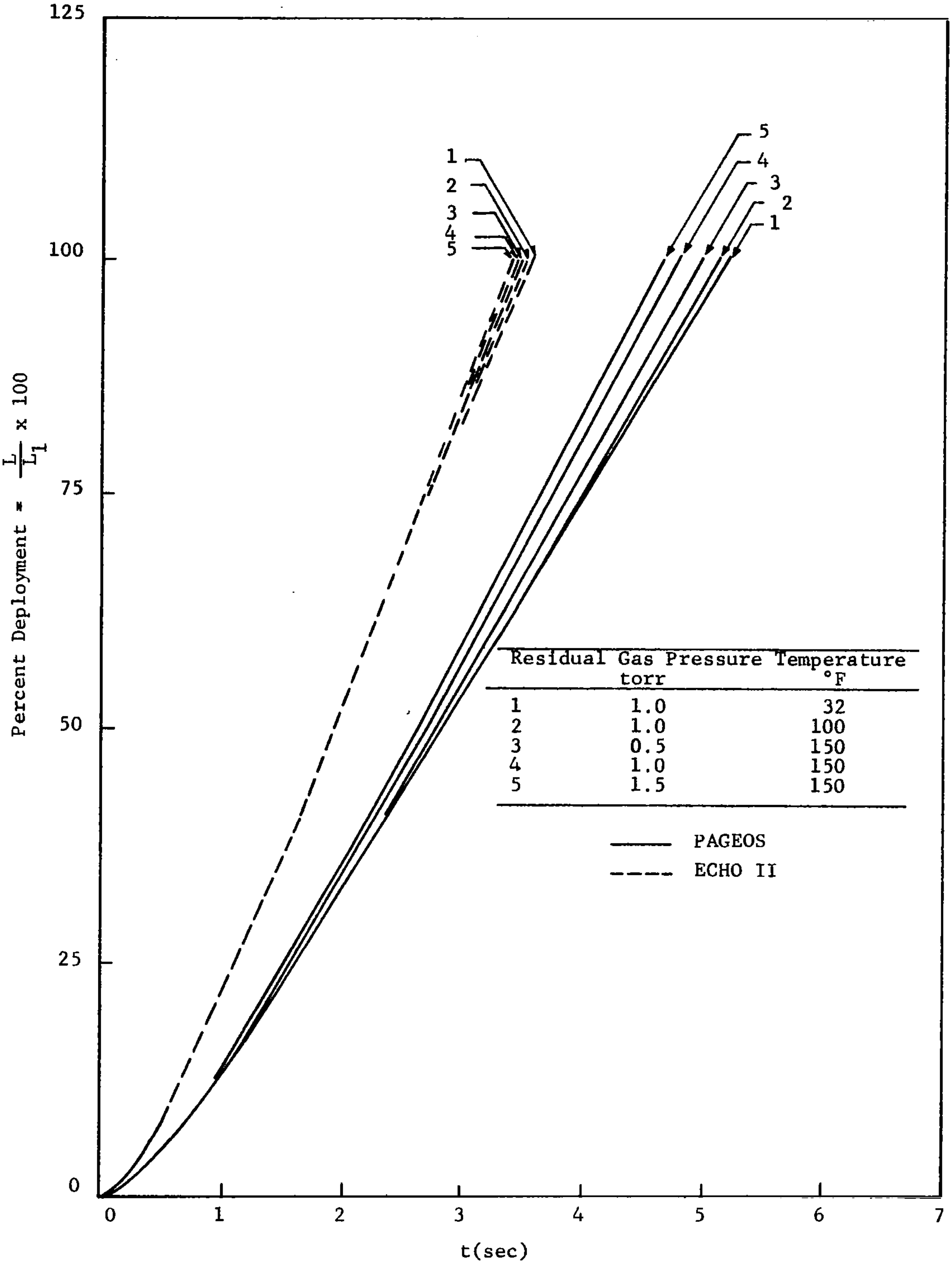


Figure 17 Deployment versus Time

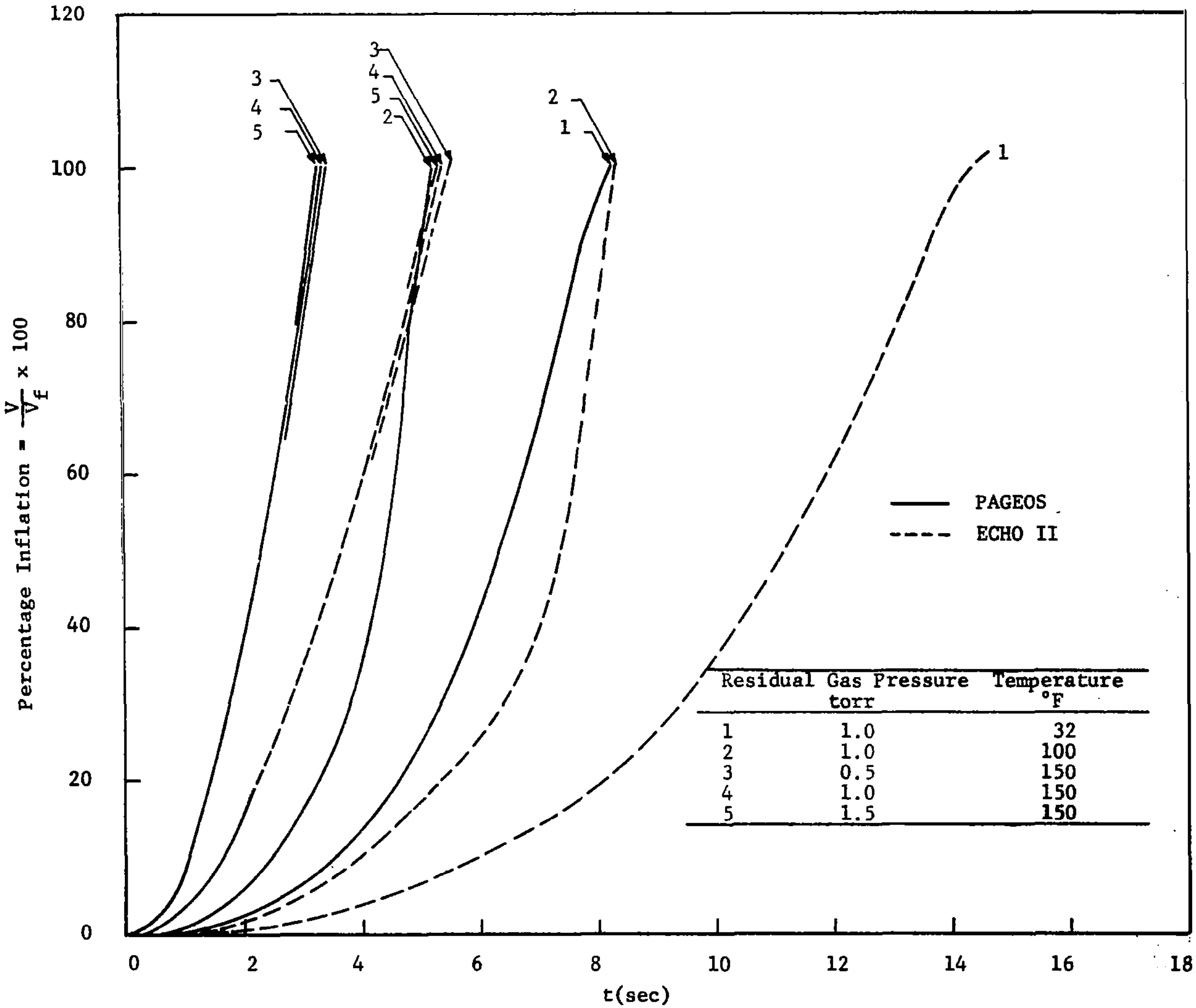


Figure 18 Inflation versus Time

Table II

TIP VELOCITY AND REFERENCE STRESS AT THE END OF THE DEPLOYMENT STAGE

	PAGEOS					ECHO II				
Residual gas pressure(torr)	1.0	1.0	0.5	1.0	1.5	1.0	1.0	0.5	1.0	1.5
Temperature (°F)	32.0	100.0	150.0	150.0	150.0	32.0	100.0	150.0	150.0	150.0
$\dot{L}_i(t^t/s_c)$	16.99	17.33	19.27	19.27	20.29	33.32	33.46	34.35	34.54	34.74
$\sqrt{\frac{ME}{\pi h_s}}\,\frac{\dot{L}_i}{D_1}$ (psi)	2152.6	2195.7	2441.5	2507.4	2590.7	9766.0	9807.1	10068.0	10123.7	10182.3

Table III

MAXIMUM STRESSES AFTER THE END OF INFLATION
(Absolute Maxima)

	PAGEOS					ECHO II				
Residual gas pressure(torr)	1.0	1.0	0.5	1.0	1.5	1.0	1.0	0.5	1.0	1.5
Temperature (°F)	32.0	100.0	150.0	150.0	150.0	32.0	100.0	150.0	150.0	150.0
σ^{o}_{mx}	967.4	2377.0	1877.3	1885.7	1894.0	1770.0	6561.8	3445.4	3443.6	3443.1
$\sigma^{\pi/2}_{\phi mx}$	836.6	2079.5	1649.5	1657.7	1665.7	1484.1	5567.6	2944.7	2943.0	2942.5
$\sigma^{\pi/2}_{\theta mx}$	1251.5	3023.2	2371.8	2380.7	2389.7	2327.2	8499.1	4421.2	4419.1	4418.6

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